Abstract—We put forward Adaptive PLE (APLE) based on Strong Tracking Filter (STF) for bearing and frequency measure in order to rectify the bias of PLE. Furthermore, the APLE avoids filter divergence in that it is not necessary to linearize non-linear measure equation. Simulation demonstrate that the APLE has the ability to reduce estimate bias adaptively and estimate accuracy can be greatly improved.

Keywords— target motion analysis; strong tracking filter; adaptive filtering; passive tracking;

I. INTRODUCTION

The basic problem of target motion analysis (TMA), as a kind of classical nonlinear estimation problems, is to estimate the relative position and velocity between target and observer from passive noise-corrupted measurement, such as sonar/frequency, Infrared etc, collected by a single moving observer. The usual approach for nonlinear estimation is to employ an extended Kalman filter (EKF). Because the bearing is an incomplete position observation, EKF has been shown to provide erratic estimation results and unstable behavior [1]. Therefore, design of filter has been the focus of passive tracking [1-3]. Tracking using bearings-only measure has been well investigated, while tracking with Doppler-bearing measure is a relatively recently development [4-5].

The well-known PLE [1], proposed for bearings-only tracking, has been extensively used in that it has linear kalman filter form and is easy to implement. Furthermore, PLE overcomes the divergence of EKF because it doesn’t have to linearize non-linear measure equation. But due to correlation between state and pseudo-measure noises, the PLE only supplies a biased estimate. To overcome this bias, [3,4] develop Maximum Likelihood Estimator (MLE) and instrumental variables methods that give satisfactory results. MLE is gradient-search based batch method, whose convergence is sensitive to initial conditions and step sizes. In [4], the correlation is ridded out of by instrumental variables, and a recursive, unbiased estimator is obtained. In the Doppler-bearing tracking, the measurement equation is also nonlinear, but by an appropriate choice of state vector, linear equation similar to PLE can be obtained. The same shortcoming of biased estimate is also present. The advantage of the Doppler-bearing tracking is that it doesn’t require a maneuver of the observer for observability, the only condition is that the emitted frequency must be constant and bearing measurement is not constant [6]. In this paper, in order to reduce estimate bias, we propose Adaptive PLE (APLE) based on Strong Tracking Filter (STF) proposed in [7], which has been successfully applied in many nonlinear time-varying stochastic systems [8-13] in that it has the ability to rectify the bias of estimate adaptively and track state change rapidly no matter whether the filter runs in steady state or not. The core of STF is that the filtering residual error vectors taken at different steps have been made orthogonal to each other (i.e. whitening filtering residual) adaptively through introducing the fading factor determining by orthogonality principle to compensate for effects of unmodelled disturbance. Hence, the APLE has the ability to rectify estimate bias adaptively and overcomes the shortcoming of original PLE. Moreover, APLE hold the advantages of convergence. Computer simulations demonstrate that APLE can reduce estimate bias greatly and more accurate state estimate can be achieved compared to original PLE.

II. TRACKING PROBLEM WITH BEARING AND FREQUENCY MEASUREMENT

In Cartesian coordinates, the discrete relative dynamic equation for Doppler-bearing tracking is

\[ x(k+1) = F(k)x(k) + Γ(k)v(k) − u_o(k) \]  (1)

the state vector consists of the relative position, velocity and frequency.

\[
\begin{bmatrix}
ξ(k) \\
ξ_t(k) \\
η(k)
\end{bmatrix}
= \begin{bmatrix}
ξ(k) − ξ_o(k) \\
ξ_t(k) − ξ_o(k) \\
η(k) − η_o(k)
\end{bmatrix}
\]  (2)

where ξ is the unknown emitted frequency assumed constant, ξ_o and ξ_t are the observer and target position coordinates, respectively. The process noise \( v(k) = \begin{bmatrix} v_ζ(k) \\ v_η(k) \end{bmatrix} \) is used to model unpredictable target acceleration in each coordinate assumed to be zeros mean, white and Gaussian with variance \( Q_{2x2}(k) \).
State transition matrix
\[
F(k) = \begin{bmatrix}
1 & T_k & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & T_k & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\] (4)

where \( T_k \) is the interval between \( k \) and \( k+1 \)

\[
\Gamma(k) = \begin{bmatrix}
T_k^2/2 & 0 \\
T_k & 0 \\
0 & T_k^2/2 \\
0 & T_k \\
0 & 0
\end{bmatrix}
\] (5)

The vector \( u_o(k) \) models the observer motion where
\[
u_o(k) = \begin{bmatrix}
\xi_o(k+1) - \xi_o(k)T_k - \xi_o(k) \\
\eta_o(k+1) - \eta_o(k)T_k - \eta_o(k) \\
\hat{\eta}_o(k+1) - \hat{\eta}_o(k) \\
0
\end{bmatrix}
\] (6)

Bearing measurement is \( \beta(k) = \theta(k) + w_1(k) \) (7)

Frequency measurement (Doppler shifted)
\[
\gamma(k) = \gamma \left( 1 - \xi(k) \sin \theta(k) + \eta(k) \cos \theta(k) \right) + w_2(k) \] (8)

where \( c \) is the velocity of sound in the medium,
\[
\theta(k) = \tan^{-1} \left( \frac{\xi(k)}{\eta(k)} \right)
\] (9)

The total measurement is
\[
z(k) = \begin{bmatrix}
\beta(k) \\
\gamma(k)
\end{bmatrix} + w(k)
\] (10)

Where \( w(k) = \begin{bmatrix} w_1(k) & w_2(k) \end{bmatrix} \) measurement noise is assumed to be white Gaussian, zeros mean with variance
\[
R(k) = \begin{bmatrix}
\sigma_1^2(k) & 0 \\
0 & \sigma_2^2(k)
\end{bmatrix}
\] (11)

III. MEASUREMENT TRANSFORMATION

Define new state vector
\[
x_o(k) = \begin{bmatrix}
\xi(k) \\
\xi(k) \\
\eta(k) \\
\eta(k) \\
\gamma(k)^{-1}
\end{bmatrix}
\] (12)

Bearings pseudo-measurement:
\[
0 = H_o(k)x_o(k) + \epsilon_o(k)
\] (13)

where
\[
\epsilon_o(k) = [\xi(k) \sin \beta(k) + \eta(k) \cos \beta(k)] \tan w_1(k)
\approx [\xi(k) \sin \beta(k) + \eta(k) \cos \beta(k)]w_1(k)
\] (13)

\[
H_o(k) = [\cos \beta(k) \ 0 \ - \sin \beta(k) \ 0 \ 0]^	op
\] (14)

According to (7-8),
We obtain:
\[
1 = \gamma(k)\gamma^{-1} + \xi(k)\sin \theta(k) + \eta(k)\cos \theta(k) - w_{z_2}(k)
\] (15)

Total measurement transformation is:
\[
z_{r}(k) = \begin{bmatrix}
0 \\
1
\end{bmatrix} = H(k)x_o(k) + w_{r}(k)
\] (16)

where \( H(k) = [H_1(k) \ H_2(k)] \) (17)

\[
w_{r}(k) = [\epsilon_1(k) \ \epsilon_2(k)]^	op
\] (18)

whose covariance is \( R_{r}(k) \)

IV APLE Recursive Formulation based on STF

For brief, the APLE Recursive Formulation are listed directly:
1. Initialization \( \hat{x}_d(0 | 0), P(0 | 0) \) (19)
2. State prediction \( \hat{x}_d(k+1 | k) = F(k)\hat{x}_d(k | k) - u_o(k) \) (20)
3. Determining the fading factor
\[
lmd(k+1) = \text{diag}[\lambda(k+1) \lambda(k+1) \cdots \lambda(k+1)]
\] (21)

\[
\lambda(k+1) = \begin{cases}
c(k+1); & c(k+1) > 1, \\
1; & c(k+1) \leq 1,
\end{cases}
\] (22)

\[
c(k+1) = \frac{tr[N(k+1)]}{tr[M(k+1)]}
\] (23)

\[
N(k+1) = V_o(k+1) - \beta R_{r}(k+1)
\] (24)

\[
M(k+1) = H(k+1)F(k)P(k | k)F(k)H(k+1) + \lambda(k+1)
\] (25)

\[
V_o(k+1) = E[\gamma(k+1)\gamma^{-1}(k+1)]
\]
where $\rho$ is a forgetting factor, which can be determined by Monte Carlo simulation, $0 < \rho < 1$, $\beta \geq 1$ for smooth the state estimate.

4. State covariance prediction:

$$P(k+1|k) = \text{F}(k)P(k|k)\text{F}^T(k)$$

$$+ \Gamma(k)Q(k)\Gamma^T(k)$$

(27)

5. Determining gain matrix

$$G(k+1) = P(k+1|k)H^T(k+1) \times$$

$$[H(k+1)P(k+1|k)H^T(k+1) + R_p(k+1)]^{-1}$$

(28)

5. State update

$$\hat{x}_d(k+1|k+1) = \hat{x}_d(k+1|k) - G(k+1)$$

$$\times [z_f(k+1) - z_f(k+1|k)]$$

(29)

6. Covariance update

$$P(k+1|k+1) = [I - G(k+1)H(k+1)]P(k+1|k)$$

(30)

IV. SIMULATION

Our simulation scenario is taken from [4], the observer is stationary at the origin and the target commences at initial (-1.8, 20km km) and proceeds due East at 9m/s radiating a single 300Hz tone. The bearings and frequency measurements Root Mean Square Error (RMSE) are $\sigma_1 = 1^\circ$ and $\sigma_2 = 0.3$ Hz. The measurement interval $T = 5s$ and there is a total 300 measurements at the end of the run. Because the observer is stationary, the $u_o(k)$ can be omitted in this tracking scenario.

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