Pattern Beamformer of Cylindrical Conformal Array

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Abstract—The minimum variance distortionless response (MVDR) beamforming optimization method is studied on cylindrical conformal array. non-isotropic antenna element pattern is used for practical engineering. Synthesis examples of three-dimensional cylindrical conformal arrays are presented. Moreover, we show by means of simulations that the method is applicable to cylindrical conformal array.

Keywords-pattern beamformer; conformal array; array signal processing

I. INTRODUCTION

The analysis and design of conformal arrays have received considerable attention because of their flexibility in attaching to arbitrary surface of vehicles and aircrafts to save space and capability of offering wide angular coverage and avoiding boresight error, etc [1]. However, conformal arrays also present many challenges to the designers, such asmanifesting themselves with different element patterns and orientations because of the varying curvature.

Beamforming is one such technique that is the use of adaptive or smart antennas to produce a movable beam pattern, which can be directed to the desired coverage areas and minimize the impact of unwanted noise and interference, thereby improving the quality of desired signal. Conventional approaches of Beamforming for ULA are relatively mature, such as least mean square(LMS) algorithm[2], recursive least square(RLS) [3] algorithm, MVDR[4][5] algorithm, and other combined algorithms[6]. The LMS or RLS are two commonly used algorithms for beamforming. The former has good tracking performance with low computational complexity, and is robust against numerical errors [2]. On the other hand, the RLS algorithm can achieve a faster convergence that is independent of the engine-value spread variations of the covariance matrix [7]. MVDR beamformer is one of the key algorithms in signal processing. In this paper, we will show that MVDR can be used for directional elements in three-dimensional cylindrical conformal array as well.

II. CONFORMAL ARRAY DATA MODEL

In this section we consider a $3 \times 3$ cylindrical conformal array shown in Fig.1.

The far field of a system of simultaneously excited cylindrical conformal array of antennas may be written, omitting the time phase factor, as

$$F(\varphi, \theta) = \sum_{i=1}^{N} \sum_{j=1}^{K} a^*_{i,j} f_i(\varphi, \theta_j) e^{-j \varphi_{i,j}}$$

Through Euler rotation matrix [8] [9], and we consider that the pattern function of each element is

$$f_i(\varphi, \theta_j) = \cos \theta_j$$

we can rewrite (1) as

$$F(\varphi, \theta) = \sum_{i=1}^{N} \sum_{j=1}^{K} a^*_{i,j} e^{-j \varphi_{i,j}} e^{2\pi j \sin \theta \cos(\varphi - \varphi_i) + \cos \theta}$$

III. OPTIMIZATION METHOD FOR CONFORMAL ARRAY BEAMFORMER

Consider a cylindrical conformal array consisting of $M$ antennas. Assuming that the $K$ arriving signals are
narrowband signals, we denote the array steering vector as
\[
\mathbf{g}(\theta) = \left[1 \ e^{-j\theta_1} \ \ldots \ e^{-j(M-1)\theta_1}\right]^T
\]  
We know the output of linear array at time \( n \) is given by
\[
y(n) = \mathbf{w}^H \mathbf{x}(n)
\]  
where \( \mathbf{x}(n) = [x_1(n) \ \ldots \ x_M(n)]^T \in \mathbb{C}^M \) is the array observation vector, \( \mathbf{w} = [w_1(n) \ \ldots \ w_M(n)]^T \in \mathbb{C}^M \) is the complex vector of beamformer weights. and \((\cdot)^H\) stands for the Hermitian transpose.

The observation vector can be written as
\[
\mathbf{x}(n) = \mathbf{s}(n) + \mathbf{i}(n) + \mathbf{v}(n)
\]  
where \( \mathbf{s}(n) \), \( \mathbf{i}(n) \), and \( \mathbf{v}(n) \) are the desired signal, interference, and noise components, respectively. \( \mathbf{v}(\theta) \) is the presumed desired signal steering vector. \( \mathbf{s}(n) \) is the desired signal waveform. The desired signal and interferers are assumed to be uncorrelated.

The MVDR beamformer \([10]\) minimizes the output interference-plus-noise power while maintaining a distortionless response to the desired signal. The MVDR problem is given by
\[
\min \|\mathbf{w}\|^2 \quad \text{st.} \quad \mathbf{w}^H \mathbf{g}(\theta) = 1
\]  
\[
\mathbf{R} = E[\mathbf{x}(n) \mathbf{x}^H(n)]
\]  
\[
= \mathbf{A} E[\mathbf{s}(n) \mathbf{s}^H(n)] \mathbf{A}^H + E[\mathbf{v}(n) \mathbf{v}^H(n)]
\]  
In many cases, \( f(\theta) \) satisfies the conditions that \( f(\theta_k) \neq 0 \) (1 \( \leq k \leq K \)) and the family of vectors \( f(\theta) \mathbf{v}(\theta) \) (1 \( \leq k \leq K \)) is linearly independent. \( \mathbf{R} \) is a positive definite matrix all the same.

The closed-form solution to (6) is
\[
\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}^{-1} \mathbf{v}(\theta)}{f^T(\theta) \mathbf{v}(\theta)}
\]  
IV. SIMULATION

In the experiment, the performance of MVDR beamformer versus omnidirectional elements for 3\( \times \)3 cylindrical conformal array is studied.

The simulation setup is as follows: 3\( \times \)3 cylindrical conformal array with 3 elements spaced at a half wavelength apart in the same ring, and there are 3 rings spaced at a half wavelength apart too (as \( L = \lambda/2 \) shown in Fig.1). The additive noise plus interference is assumed to be a zero mean complex Gaussian noise. We use 512 snapshots.

There are three signals impinging upon all arrays:
1) the signal of interest \( s(t) \) with angle of arrival \( \phi_s=20^\circ, \theta_s=60^\circ \).
2) the first interference signal \( i_1(t) \) with angle of arrival \( \phi_1=-40^\circ, \theta_1=40^\circ \).
3) the second interference signal \( i_2(t) \) with angle of arrival \( \phi_2=60^\circ, \theta_2=80^\circ \).
4) the third interference signal \( i_3(t) \) with angle of arrival \( \phi_3=0^\circ, \theta_3=130^\circ \).
5) the fourth interference signal \( i_4(t) \) with angle of arrival \( \phi_4=-10^\circ, \theta_4=90^\circ \).

The received noise variance \( \sigma^2 = 1 \) and the amplitude of \( s(t), i_1(t), i_2(t) \), and \( i_3(t) \) is determined based on the SNR and INR.

![Beam Pattern of 3\( \times \)3 Cylindrical Conformal Array](image1)

![Contour Plot of 3\( \times \)3 Cylindrical Conformal Array](image2)
The simulation result is shown in Figures above. It can be seen that in the sidelobe region the optimum beam pattern has a deep notch at the locations of interferers and the other lobes are well behaved. The MVDR beamformer for directional elements outperforms in terms of the main beamwidth and and sidelobe level. In contrast, sidelobe structure in elevation shows differences; the reason is that the directional element has different gain as the angle changes. At the locations of interferers, the depth of the directional elements is about -60dB, which is deeper than the omnidirectional. The beamformer of directional elements converges more rapidly if the jammer arrives in that area.

V. CONCLUSION

A MVDR beamformer for cylindrical conformal array of directional elements is proposed. The analysis method proposed in this paper is suitable not only for the cylindrical conformal array, but also for other conformal array.

Simulations indicate that this method is an effective tool for practical conformal array beamforming.

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