Theoretical Study on Time Delay and Doppler stretch estimation of Chirp signal based on Wavelet-Cumulants

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Abstract—The algorithm of wavelet analysis can be used to estimate the time delay (TD) and Doppler stretch (DS) of Chirp signals, but it cannot suppress the effect of Gaussian noises. In this paper, a definition of fourth-order Wavelet-Cumulants is given by combining the wavelet and the forth-order cumulant. This method can be used in unknown Gaussian noise environment. The simulation results demonstrate the effectiveness of this novel approach.

Keywords—Wavelet-Cumulants; Chirp signal; Time Delay; Doppler stretch

I. INTRODUCTION

In the time-variant signals, the Chirp signal is a common signals signal, but has the representation of non-stationary signal[1]-[3]. As a great time-bandwidth product of the spread-spectrum signal, Chirp signal is widely used in various information systems, such as communications, radar[4], geophysics[5] and seismic exploration, etc.; at the same time, when the target to accelerate and target Doppler frequency in detection systems is approximately proportional to target velocity, the echo could look as the Chirp signal. In recent years, the study focuses on such signals, mainly because the echo signals will be delayed, and the frequency will also be changed, when the objectives move relative to the receiving transducer. In order to get distance and speed information at the same time, it should estimate a joint time delay and Doppler. Therefore it will estimate time delay and Doppler parameters of the Chirp signal.

The outstanding difficulty of chirp parameter estimation is that chirp signal is a typical nonstationary signal. In recent years, wavelet transform has been used to analye the time-varying frequency content of nonstationary signals [6]. Attention has been focused on using this transform to analyze Chirp signal [7]. The property of the signal could be analyze because of window functions. So we consider combining the wavelet transform with high-order cumulants, which are powerful to denoise in the background of the coloured Gaussian noise[8]. Therefore, we propose a new algorithm : Wavelet-Cumulants.

II. SIGNAL MODELS

The discrete single-Chirp signal is expressed as:

\[ x(t) = \alpha e^{j2\pi \left( \frac{k}{2} t^2 + f_0 t \right)} \] (1)

Where \( \alpha \) is the phase of the Chirp signal, and \( f_0 \) is the initial frequency of the Chirp signal and \( k \) is FM slope.

The corresponding single-component Chirp signal is as follow [9]:

\[ y(t) = \beta s(t - \tau_0) + \omega(t) \] (2)

Similarly, since \( x(t) \) is even,

\[ y(t) = \beta e^{j2\pi \left( \frac{k}{2} t^2 - \tau_0 \right)} + \omega(t) \] (3)

Where \( \beta \) is the attenuation coefficient, and the unkowned parameter \( \tau_0 \) is time delay, \( \sigma \) is Doppler stretch. \( \omega(t) \) is the Gaussian noise.

III. WAVELET-CUMULANTS

Wavelet transform has the ability of localizing a signal in both time and frequency. This capability of wavelet transform is explored in the detection of the Chirp signals. And Higher-order cumulants are very powerful tools to denoise.

Set up \( f(t) \), \( f_1(t) \), \( f_2(t) \), \( f_3(t) \) signals have time-average, fourth order Wavelet-Cumulants defined as

\[ W_{4,4,4,4}(\tau, \sigma; \tau, \sigma; \tau, \sigma; \tau, \sigma) \]

\[= \frac{1}{\sqrt{\sigma \sigma \sigma \sigma}} \text{cov} \left( f(t), f_1(t), f_2(t), f_3(t) \right) \]

In the form of time-average, we have that

\[ W_{4,4,4,4}(\tau, \sigma; \tau, \sigma; \tau, \sigma; \tau, \sigma) = \frac{1}{\sqrt{\sigma \sigma \sigma \sigma}} \text{cov} \left( f(t), f_1(t), f_2(t), f_3(t) \right) \]

\[= \frac{1}{\sqrt{\sigma \sigma \sigma \sigma}} \text{cov} \left( f(t), f_1(t), f_2(t), f_3(t) \right) \]

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When the signal \( x(t) \) include the noise, by nature of fourth-order Wavelet-Cumulants\[9\], allows (1) to be rewritten as

\[
W_{xy}(\tau, \sigma) = W_{xy}(0, 1; 0, 1; \tau, \sigma) = W_{xx}(0, 1; 0, 1; \tau, \sigma) + W_{w}(0, 1; 0, 1; \tau, \sigma) \tag{6}
\]

Where \( x = s \), \( x \) is the discrete single-Chirp signal, \( y = r + w \), \( y \) is the corresponding single-component signal, \( w \) is noise.

The same to use the nature of fourth-order cumulants, any Gaussian processing equate to zero, so equation (6) can be written as follows:

\[
W_{xy}(\tau, \sigma) = W_{xy}(0, 1; 0, 1; \tau, \sigma) \tag{7}
\]

Chirp signal is zero-mean, so fourth-order Wavelet-Cumulants definition of Chirp signal can express in the equation (7), and the additive noise is a smooth set of ergodic and zero-mean Gaussian noise, so we received Chirp signal fourth-order Wavelet-Cumulants as follows:

\[
W_{xx}(\tau, \sigma) = W_{xx}(0, 1; 0, 1; \tau, \sigma)
\]

\[
W_{xy}(\tau, \sigma) = W_{xy}(0, 1; 0, 1; \tau, \sigma)
\]

\[
W_{xx}(\tau, \sigma) = W_{xx}(0, 1; 0, 1; \tau, \sigma)
\]

Finally, as formula (8) implies that the algorithm of Wavelet-Cumulants can estimate time delay and Doppler stretch parameter. So this algorithm increases the analysis ability of the Chirp and suppressing Gaussian noise.
Figure 2. The estimation of TD and the DS of the Chirp signal in Wavelet-Cumulants.

In Fig. 1 the algorithm we used is the wavelet, and in Fig. 1 the algorithm is the Wavelet-Cumulants. Fig. 1 and Fig. 2 shows that the peak is value of the time delay and Doppler stretch \( \tau = 0.11 \), \( \sigma = 1.0030 \) in Fig. 1, and \( \tau = 0.227 \), \( \sigma = 1.0190 \) in Fig. 2. At the same time the simulation is done in the Chirp signal in the background of non-noise. The peak expresses the energy of the Chirp signal. In contrast to Fig. 1, the peak of Fig. 2 is very sharp and high, so we get that the estimation of the Chirp signal is accurate in Wavelet-Cumulants. Through Fig. 1 the peak is not very sharp, so we are difficult to estimate the parameters. Therefore, this algorithm embodies its advantage comparing with the algorithm of wavelet.

Fig. 1 and Fig. 2 is the result in non-noise, we don’t know this algorithm is very useful in Gaussian noise. Following simulation is in white Gaussian noise.

A. The joint estimation of TD and the DS of the Chirp signal in white Gaussian noise

Figure 3. The joint estimation of TD and the DS of the Chirp signal in wavelet in white Gaussian noise.
Fig. 4 indicates that we can estimate the exact values of time delay and Doppler stretch coefficient of the Chirp signal in the background of white Gaussian noise, compared with Fig. 3. The main peak is still more obvious, and pseudo-peak power is relatively low. When the signal-to-noise ratio (SNR) is added gradually, the estimation is not influenced. The results show that the value is estimated accurately with suppressing the effect of Gaussian noises. Through above discussion, the algorithm can get an accurate estimation results.

Fig. 5. The joint estimation of TD and the DS of the Chirp signal in wavelet in colored Gaussian noise.
When environment noise is colored Gaussian noise, simulation results are shown in Fig. 6. The main peak is still more obvious, and do not be smaller with the increasing SNR. So we also can estimate the value of TD and DS. In Fig. 2, the results show that with estimation accuracies of time delay and Doppler parameters, $\tau = 0.227$, $\sigma = 1.0190$. In Fig. 4 and Fig. 6, the estimated results and the simulation results are consistent with the result in Fig. 1. From Fig. 2, Fig. 4 and Fig. 6 we can see that sharp level of peak is the same, the power pseudo-peak is relatively low, and the figure reflects higher distinguishability, thus embodies the advantages of Wavelet-Cumulants. And in the same circumstances, distinguishability in the background of white Gaussian noise is better than that in the background of Colored Gaussian noise.

V. CONCLUSION

In the present paper we have analyzed the advantage of wavelet analysis and cumulant, at the same time we detect the shortage of them. Combining with the advantage of wavelet analysis and cumulant, we define a new algorithm — Wavelet-Cumulants. The method is generalized by enhance the ability of the signal analysis and increasing the ability of suppressing noise. So the algorithm of Wavelet-Cumulants includes the advantage of wavelet analysis and cumulant. This algorithm has a stronger capability of signal analysis, while the result shows that a joint time delay and Doppler parameters can be estimate accurately and the Gaussian noise would be reduced when the Gaussian noise be added to the signal. The algorithm of Wavelet-Cumulants can reflect the change of Chirp signal in detail through estimating a joint time delay and Doppler parameters accurately. So this algorithm increases the ability of the Chirp analyst and suppressing Gaussian noise.

REFERENCES