Compensation of Hysteresis Nonlinearity for the Piezoelectric Actuators

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Abstract—These Adaptive inverse compensation is adopted to eliminate the hysteresis nonlinearity of piezo-actuator in active control system. The adaptive control law based on grads method is obtained with Prandtl-Ishlinskii model. It is proved that when Prandtl-Ishlinskii model is used, results of compensation have no contact with slopes under the assumption that backlash model has a symmetry structure and backlashes as well as slopes are all proportional with each other. As only proportion of reference input and output is related with slopes, the estimated parameters are reduced to only one. The results of simulation indicated that compensation effect of adaptive inverse control to hysteresis nonlinearity was much better when signal frequency was low.

Keywords—Piezo-actuator; Hysteresis compensation; Adaptive inverse control

I. INTRODUCTION

There is an increasing usage of piezoelectric actuators in precision machining and vibration active control, because they have fast response, high stiffness, and no friction. A piezoelectric actuator, however, exhibits highly nonlinear behavior that can only be described well by a nonlinear model. The most significant nonlinearity of a piezo-actuator is in the form of hysteresis. Additionally, its behavior also nonlinearly depends on the frequency of input. Therefore, the performance will be poor without compensating for the hysteresis and frequency dependent behavior of the piezo-actuator in use.

It is customary, in the compensation of hysteresis, to obtain a model of the hysteresis plant, and then this is part of the system controller in order to compensate for hysteresis. P.Ge et al.[1] used a combination of a PID feedback controller and a feed-forward compensator based on the classical Preisach model to reduce hysteresis in a piezo-actuator. Galinaitis et al.[2] used an Inverse classical Preisach model based control architecture to compensate the hysteresis effect in an unloaded piezoelectric stack actuator. Attempts have also been made to use artificial neural networks to compensate for hysteresis in mechanical and piezo-actuated systems[3,4]. Adaptive control has also been used to control plants that exhibit hysteresis behavior[5,6]. All of the above mentioned studies have focused on the compensation of hysteresis under static excitation condition, and are therefore not suitable for dealing with nonlinearities.

Adaptive Inverse Control was proposed by B.Widrow in 1986. Adaptive filtering algorithm, and then is placed at the input as the controller to control the dynamical characteristics of the control object.

Considering the nonlinear systems, the inverse model of control object can be used as controller. It can adjust its parameters adaptively so that the input has linear relation with output of the system. The nonlinearity can be compensated by this way. Whether the method of adaptive inverse control can be applied totally lies on the existence and accuracy of inverse model.

Control systems can work because they have their own actuators for instance motor, valve, electromagnetism actuator, piezoelectricity component, magnetostrictive actuator and so on. These actuators often have non-smooth and nonlinear characteristics just as dead-zone, backlash, hysteresis and so on.

In this paper, Adaptive inverse compensation is adopted to eliminate the hysteresis nonlinearity of piezo-actuator in active control system. The adaptive control law based on grads method is obtained with Prandtl-Ishlinskii model. It is proved that when Prandtl-Ishlinskii model is used, results of compensation have no contact with slopes under the assumption that backlash model has a symmetry structure and backlashes as well as slopes are all proportional with each other. As only proportion of reference input and output is related with slopes, the estimated parameters are reduced to only one.

Figure 1. Electromechanical model of piezo-Actuators: Electric Model
(b) mechanical model

Figure 1.
II. ELECTROMECHANICAL MODEL OF PIEZO-ACTUATORS

In order to simplify the mechanical structure of piezo-actuator, it can be expressed to be mass spring damper. Paper[7] proposed an accurate electromechanical model of piezoelectric actuator, see Fig.1.

Hysteresis and piezoelectricity effects are considered separately. \( T_{en} \) is used to describe piezoelectric effect, equivalent to a electromechanical converter with conversion rate of \( T_{en} \). \( C_p \) denotes the sum of the parallel capacitance of the piezoelectric film. The total flow circuit current \( i = \frac{q}{g6} \).

\( q \) is the total charge of parallel piezoelectric actuator. \( \text{emT} \) is the conversion charge from the mechanical side. Voltage \( \text{p} u \) is caused by the piezoelectric effect. The total voltage input of piezoelectric actuator is \( u_{in} \).

\( f_{ext} \) is the force that external structure is counterproductive to the piezoelectric actuator. \( x \) denotes the output displacement of actuator. The relationship between \( f_{ext} \) and \( x \) is established through mass \( m_a \).

According to the principle of conservation of mechanical energy and energy, we have \( u_a q_p = f_{ext} x \).

Piezoelectric actuator electromechanical model is described by the following equation:

\[
\begin{align*}
  u_p &= u_m - u_h \\
  u_h &= H(q) \\
  q &= C_p u_p + q_p \\
  q_p &= T_{en} x \\
  f_a &= T_{en} u_p \\
  m_a \ddot{x} + c_v \dot{x} + k_v x &= f_a - f_{ext} \\
\end{align*}
\]

(1)

The hysteresis non-linearity of the actuator is modeled by first dividing the dynamics of the whole structure into linear and non-linear parts, in cascade form, as illustrated in Fig. 2.

The mechanical part of this actuator and the linear terms of the piezoelectric stack are considered to be second order linear dynamic. The proposed model of the piezoelectric actuator combines the second order dynamics with the cascaded hysteresis non-linearity for the piezoelectric actuator.

![Figure 2. Simplified model of piezoelectric actuator](image)

III. PIEZO-ACTUATOR HYSTERESIS MODEL AND THE INVERSE MODEL

Prandtl-Ishlinskii model is used to build the piezoelectric actuator hysteresis and the inverse model.

A. Hysteresis model

Prandtl-Ishlinskii model is built by several backlash models in the form of weighted superposition. Backlash model can be expressed as follows:

\[
\begin{align*}
  u(t) &= B(v(t)) = \begin{cases} 
    m(v(t) - r) & \text{if } v(t) \leq v_r \\
    m(v(t) - r) & \text{if } v(t) \geq v_r \\
    u(t-1) & \text{if } v_l < v(t) < v_r \\
  \end{cases}
\end{align*}
\]

(2)

Where,

\[
\begin{align*}
  v_l &= \frac{u(t-1) + r_l}{m} \\
  v_r &= \frac{u(t-1) + r_r}{m}
\end{align*}
\]

\( r_r \) is the right “crossing”, while \( r_l \) is the left “crossing”. The slope is \( m \).

The hysteresis model based on Prandtl-Ishlinskii model can be obtained by the equation as follows:

\[
H[v](t) = \sum_{i=0}^{n} m_i \cdot H[v, u_{in}](t) = m^T \cdot H[v, u_{in}](t)
\]

(3)

Where \( u_{in} = (u_{in1} \cdots u_{inn}) \) represents initial value vector and \( m^T = (m_1 m_2 \cdots m_n) \) is the weight vector. \( v \) is the input. While \( r^T = (r_l r_r \cdots r_r) \) is the backlash width vector, its value usually be selected according to the following equation:

\[
\begin{align*}
  r_i &= \frac{1}{n+1} \max \{|v(t)|, \quad i = 0 \text{ to } n \} \\
\end{align*}
\]

(4)

B. Hysteresis Inverse Model

The hysteresis model based on Prandtl-Ishlinskii model can be obtained by the equation as follows:

\[
H^{-1}[u](t) = \sum_{i=0}^{n} m_i^T \cdot H[H[v], u_{in}](t)
\]

(5)

Where \( m^T = (m_1^T m_2^T \cdots m_n^T) \) is the weight value of inverse model. Inverse model parameters are determined by the following equations:
\[ m'_0 = \frac{1}{m_0}; \]
\[ m'_i = -\frac{m_i}{(\sum_{j=0}^{n} m_j)(\sum_{j=0}^{n} m_j)}; \quad i = 1, 2, \ldots, n; \]
\[ r'_i = \sum_{j=0}^{n} m_j(r'_j - r'_i); \]
\[ u'_i = \sum_{j=0}^{n} m_{u_0} + \sum_{j=0}^{n} m_{u_0}; \quad i = 0, 1, \ldots, n; \]

(6)

C. Inverse Model Control

Hysteresis inverse compensation principle is that hysteresis inverse model is placed at the input of hysteresis as the controller to offset hysteresis characteristics. The relationship between \( H(v(t)) \) and \( H^{-1}[u(t)] \) is called mapping and mapped according to the inverse system theory. The model and inverse model are equivalent to a standardized mapping of the unit. The control structure of piezoelectric actuator hysteresis inverse compensation shows in Fig.3.

![Figure 3. Hysteresis inverse compensation](image)

Hysteresis characteristics of piezoelectric actuator are actually not fixed but changing with the frequency of input signal and temperature and so on in practice. Adaptive estimation has been used to estimate the model parameters online. Then we designed the inverse model controller.

IV. ADAPTIVE INVERSE CONTROL

According to the principle of hysteresis inverse compensation, the structure of control system is shown in Fig.4.

![Figure 4. Structure of Control System](image)

Where \( G(D) \) is the linear part of piezoelectric actuator either in continuous time (when \( D = s \) denotes either the Laplace transform variable or the time differentiation operator \( x(t) = x(t) \)) or in discrete time (when \( D = z \) denotes either the z-transform variable or the time operator \( x(t) = x(t + 1) \)). Reference model \( G_s(D) \) is piezoelectric actuator ideal dynamic model. \( G_c(D) \) is the closed-loop linear controller.

A. Adaptive Law

Gradient method is used to design adaptive law, and then the hysteresis nonlinear model parameters are estimated online. Introducing the indicator function \( \chi[X] \) of the event X:

\[ \chi[X] = \begin{cases} 1, & \text{if } X \text{ is true} \\ 0, & \text{otherwise} \end{cases} \]

We define the backlash indicator functions:

\[ \chi_i(t) = \chi[\dot{v}(t) > 0] \]
\[ \chi_i(t) = \chi[\dot{v}(t) < 0] \]
\[ \chi_i(t) = \chi[\dot{v}(t) = 0] \]

(8)

While the expression of backlash non-linear model is:

\[ u(x) = \chi_i(t)(m(v(t) - r_i)) \]
\[ + \chi_i(t)(m(v(t) + r_i)) + \chi_i(t)(u(t - 1)) \]

(9)

According to Prandtl-Ishlinskii model, the hysteresis non-linear model is:

\[ u(t) = \sum_{i=0}^{n} \chi_i(t)(m_i(v(t) - r_i)) \]
\[ + \chi_i(t)(m_i(v(t) + r_i)) + \chi_i(t)(u(t - 1)) \]

(10)

Hysteresis non-linear inverse model is:

\[ v(t) = \sum_{i=0}^{n} \chi_i(t)(m'_i(u(t) - r'_i)) \]
\[ + \chi_i(t)(m'_i(u(t) + r'_i)) + \chi_i(t)(v(t - 1)) \]

(11)

According to Fig.7, the ideal situation is:

\[ y(t) = y_s(t) = G_s(D)u_s(t) \]

(12)

Where \( G_s(D)u_s(t) \) denotes the input of \( G_s(D) \) is \( u_s(t) \).

Such that

\[ y(t) = G(D)\sum_{i=0}^{n} \chi_i(t)(m_i'((u_s(t) - r_i') + \chi_i(t)(m'_i(u_s(t) + r_i')) + \chi_i(t)(v(t - 1)) - r_i')) \]
\[ + \chi_i(t)(m_i(u_s(t) - r_i')) \]

(13)

Thesis[15] has proved the hysteresis nonlinear compensation has nothing to do with the parameter \( m \).
In order to facilitate the derivation, we choose the backlash:

\[ r_j = -r_j' = r, \quad r'_{j} = -r'_{j} = r'. \quad (14) \]

Cost function is:

\[ J(r') = \frac{\epsilon^2}{2} \quad (15) \]

Where,

\[ \epsilon = y_{o}(t) - y(t) \]

So that:

\[ \epsilon = G_{a}(D)[u_{j}](t) - G(D)\left( \sum_{i=0}^{N} \left( x_{i}(t)m_{i}( \sum_{i=0}^{N} x_{i}^{'}(t)m_{i}(u_{j}(t) - r')) \right) - r' \right) + x_{b}(t)m_{b}(u_{j}(t) - r') + x_{b}'(t)(v_{c}(t-1) - r)) + x_{a}(t)(u_{j}(t) - 1)) \]  \( (16) \)

The gradient of cost function is:

\[
\nabla J(r') = \frac{\partial J(r')}{\partial r'} = -\epsilon G(D)\left[ \sum_{i=0}^{N} \left( x_{i}(t)m_{i}(-x_{i}^{'}(t)m_{i}') + x_{i}'(t)m_{i}' \right) \right] + \left( x_{b}(t)m_{b}(-x_{b}^{'}(t)m_{b}') + x_{b}'(t)m_{b}' \right) \]

Therefore, a adaptive update law for \( r'(t) \) is

\[ r'(t) = -\gamma \epsilon G(D)\left[ \sum_{i=0}^{N} \left( x_{i}(t)m_{i}(-x_{i}^{'}(t)m_{i}') + x_{i}'(t)m_{i}' \right) \right] + \left( x_{b}(t)m_{b}(-x_{b}^{'}(t)m_{b}') + x_{b}'(t)m_{b}' \right) \]

where \( \gamma \) is adaptiv gain and \( \gamma > 0 \).

V. SIMULATION EXPERIMENT RESULTS

These are the simulation results of control system using adaptive inverse compensation. The adaptive gain is \( \gamma = 0.2 \), sampling time is 0.001s, input is sinusoidal signal. Consider a plant with a hysteresis \( H(\cdot) \), in which the linear part is\(^{[6]}\)

\[ G(s) = \frac{2.5}{s^2 + 2s - 4} \]

Let the transfer function of reference model be chosen as

\[ W_{m}(s) = \frac{1}{s^2 + 3s + 2} \]

The linear controller is PID controller.

A. Open-loop Adaptive Inverse Compensation

To facilitate the simulation model the hysteresis model is made up of four backlash operators and so is the inverse hysteresis model. What should be stressed is the more backlash operators we use to construct the hysteresis model, the more accurate model we can get, but the inverse model will be very fuzzy in the realization. Fig.5 shows the given hysteresis curve of piezoelectric actuator. The result after compensation is shown in Fig.6. Fig.7 is the convergence curve of the estimated parameter \( r' \). The estimated value stabilized at 40 after about 0.1 seconds (The true value of \( r \) is 40). Fig.8 shows the output of piezoelectric actuator before and after using adaptive inverse compensation when the input is sinusoidal signal.

We found that when the higher the frequency of input signal becomes, the larger the parameter estimation error becomes. When the input signal is \( u_{j}(t) = 250 \sin 50\pi t \), estimated value floating in the vicinity of the ideal value. At this time the compensation effect is not good, just as Fig.11 shows.

B. Compensation with Linear Controller

Adaptive inverse control with linear controller PID could reduce the input-output error further, input and output is basically a linear relationship, the output can track input well. The results of simulation indicated the results with not only the linear controller but also the adaptive inverse controller are better than the compensation effect with only the linear PID controller.

According to the experimental results when the frequency is not too high (below), the hysteresis can be compensated nomatter the hysteresis is fixed or changing with input. Because the adaptive inverse controller could estimate the parameters of the hysteresis model and adjust the controller.

The controller with PID will enable the hysteresis loop control in less than 1%, output can be tracking the input signal well. But the shortcoming of this approach should be awared: the convergence time of estimated value is a little long and the high-frequency (above 25Hz) compensation effect is not ideal. Therefore the method can be applied to the systems that does not need high frequency, the algorithm needs to be improved.

![Figure 5. Given Hysteresis Curve](image-url)
VI. CONCLUSIONS

A common limitation of piezoceramic actuators in precision applications is hysteresis. Rate dependent nonlinear hysteresis is found in piezoceramic actuators. The main objection of the thesis is to reduce the nonlinear effect in piezoceramic actuators, making the response more precise in order to lead to high linear relationship of input and output.

Adaptive inverse compensation is adopted to eliminate the hysteresis nonlinearity of piezo-actuator in active control system. The adaptive control law based on grads method is obtained with Prandtl-Ishlinskii model. It is proved that when Prandtl-Ishlinskii model is used, results of compensation have no contact with slopes under the assumption that backlash model has a symmetry structure and backlashes as well as slopes are all proportional with each other. As only proportion of reference input and output is related with slopes, the estimated parameters are reduced to only one. The results of simulation indicated that compensation effect of adaptive inverse control to hysteresis nonlinearity was much better when signal frequency was low (less than 25Hz).

Although this thesis is based on modeling and controlling piezo-actuators, we believe that the models and the control arithmetic presented are also applicable to model and control other smart material with hysteresis phenomena.

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