A New MCDM Method Based on GRA in Interval-valued Intuitionistic Fuzzy Setting

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Abstract—In this paper the grey relational analysis (GRA) are generalized to the interval-valued intuitionistic setting. This method can be used in both the exactly known and partly known criteria weights situation, for the latter case, it is only necessary to solve one linear programming problem. A numerical example finally illustrates the practicality, flexibility and efficiency of our new algorithm.

Keywords—MCDM; IVIFS; GRA;

I. INTRODUCTION

Interval-valued intuitionistic fuzzy set (IVIFS) was introduced by Atanassov [1]. In the IVIFS, both membership and non-membership are interval numbers. They also defined the operation laws about interval-valued intuitionistic fuzzy sets. During the decision making, people often are reluctant to or can’t give accurate evaluation values, but they would like to provide an range of the evaluation value. So it is meaningful to study the decision making problem based on IVIFSs, which has now become a hot topic [2]-[4].

Grey relational analysis (GRA) method was originally developed by Deng [5] and has been successfully applied in many multiple attribute decision making problems [6]-[8]. In this method, the performance of the alternatives are translated into comparable sequence first, then the ideal target sequence is defined. The grey relational coefficient between each sequence and the ideal target sequence is calculated. The grey relational degree is calculated in the last and the alternative which has the largest grey relational degree is the best one.

In this paper, we generalize the GRA to the interval-valued intuitionistic fuzzy setting. The evaluation values of the alternatives with respect to the attributes are in IVIFNs. We develop a method to rank alternatives in completely known and partly known attribute weight information.

The rest of the paper is organized as follows: some basic concepts are illustrated in section 2. In section 3, A new method is given. A numerical example is given in section 4. In the last section, we give the conclusions.

II. BASIC CONCEPTS

Definition 2.1 [1] Let $X$ be a fixed set. An intuitionistic fuzzy set (IFS) $A$ in $X$ is defined as

$A = \{x, \mu_A(x), \nu_A(x)| x \in X\},$

where the functions $\mu_A(x): X \rightarrow [0,1]$ and $\nu_A(x): X \rightarrow [0,1]$ are the degrees of membership and nonmembership of an element $x \in X$ to the set $A$ satisfying

$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.$

Definition 2.2 [9] Let $X$ be a non-empty set of the universe, and $D[0,1]$ be the set of all closed subintervals of $[0,1]$. An interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ in $X$ is defined as an object having the following form:

$\tilde{A} = \{x, \tilde{\mu}_A(x), \tilde{\nu}_A(x)| x \in X\},$

where the functions $\tilde{\mu}_A(x): X \rightarrow D[0,1], \tilde{\nu}_A(x): X \rightarrow D[0,1]$ are the degree of belongingness and the degree of non-belongingness, respectively. And for each $x \in X$ to the set $A$ satisfying

$0 \leq \tilde{\mu}_A(x) + \tilde{\nu}_A(x) \leq 1, \forall x \in X.$

For convenience, in what follows, we denote an IVIFN by $[a,b]$, where $[a,b] \subset [0,1]$, $[c,d] \subset [0,1]$, $b + d \leq 1$.

Definition 2.3 [10] Let $\tilde{a}_1 = (\{a_1, b_1\},\{c_1, d_1\})$, $\tilde{a}_2 = (\{a_2, b_2\},\{c_2, d_2\})$ be two IVIFNs. Then the distance between $\tilde{a}_1$ and $\tilde{a}_2$ can be defined as

$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4} \left( |a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2| \right).$

III. MADM BASED ON GRA IN INTERVAL-VALUED INTUITIONISTIC FUZZY SETTING

Let $A = \{A_1, A_2, ..., A_n\}$ be a finite set of attributes, $G = \{G_1, G_2, ..., G_n\}$ be the set of attributes and $W = \{w_1, w_2, ..., w_n\}$ be the weight vector of the attribute, where $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0,1]$. Let $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ be an interval intuitionistic fuzzy
decision matrix. \([\mu^i, \mu^e_i]\) indicates the degree that the alternative \(A_i\) satisfies the attribute \(G_j\), \([v^i, v^e_i]\) indicates the degree that the alternative \(A_i\) does not satisfy the attribute \(G_j\). \(\mu^i + v^e_i \leq 1, i = 1, 2, ..., m, j = 1, 2, ..., n\).

In the following, we solve the MADM problem using GRA.

Step 1. Determine the positive-ideal solution (PIS) and negative-ideal solution (NIS) based on interval-valued intuitionistic fuzzy numbers. The following four attributes are defined as relational coefficient of each evaluation value from PIS and NIS, respectively. The grey alternative by:

\[
\begin{align*}
\tilde{r}^+ &= (r^+_1, r^+_2, ..., r^+_n) \\
&= ([\mu^{i1, \max}, \mu^{e1, \min}, [v^{i1, \min}, v^{e1, \min}], [v^{i1, \max}, v^{e1, \max}]), ..., \\
&([\mu^{in, \max}, \mu^{en, \min}, [v^{in, \min}, v^{en, \min}], [v^{in, \max}, v^{en, \max}]), \\
\tilde{r}^- &= (r^-_1, r^-_2, ..., r^-_n) \\
&= ([\mu^{i1, \min}, \mu^{e1, \max}, [v^{i1, \min}, v^{e1, \min}], [v^{i1, \max}, v^{e1, \max}]), ..., \\
&([\mu^{in, \min}, \mu^{en, \max}, [v^{in, \min}, v^{en, \min}], [v^{in, \max}, v^{en, \max}]).
\end{align*}
\]

Where \(r^+_j = ([\max \mu^{i1, \max}, \min \mu^{e1, \min}, [\min v^{i1, \min}, \min v^{e1, \min}], [\max v^{i1, \max}, \max v^{e1, \max}]), j = 1, 2, ..., n\), \(r^-_j = ([\min \mu^{i1, \min}, \max \mu^{e1, \max}, [\max v^{i1, \min}, \max v^{e1, \min}], [\min v^{i1, \max}, \min v^{e1, \max}]), j = 1, 2, ..., n\).

Step 2. Calculate the grey relational coefficient of each evaluation value from PIS and NIS, respectively. The grey relational coefficient of each evaluation value from PIS and NIS are defined as

\[
\begin{align*}
\xi^+_{ij} &= \frac{\min_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^+_i, \tilde{r}^+_j) + \rho \max_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^+_i, \tilde{r}^+_j)}{d(\tilde{r}^+_i, \tilde{r}^+_j) + \rho \max_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^+_i, \tilde{r}^+_j)}, \\
\xi^-_{ij} &= \frac{\min_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^-_i, \tilde{r}^-_j) + \rho \max_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^-_i, \tilde{r}^-_j)}{d(\tilde{r}^-_i, \tilde{r}^-_j) + \rho \max_{\{i=1,2,...,m\}} \max_{\{j=1,2,...,n\}} d(\tilde{r}^-_i, \tilde{r}^-_j)},
\end{align*}
\]

\(i = 1, 2, ..., m, j = 1, 2, ..., n\), where \(\rho \in [0, 1]\).

Step 3. Calculate the degree of grey relational coefficient of each alternative from PIS and NIS as follows:

\[
\begin{align*}
\xi^+_i &= \sum_{j=1}^{m} w_i \xi^+_{ij}, i = 1, 2, ..., m, \\
\xi^-_i &= \sum_{j=1}^{m} w_i \xi^-_{ij}, i = 1, 2, ..., m.
\end{align*}
\]

Step 4. Calculate the relative relational degree of each alternative by:

\[
\xi^+_i = \frac{\xi^+_{ij}}{\xi^+_{ij} + \xi^-_{ij}}, i = 1, 2, ..., m.
\]

Step 5. Rank alternatives according the values of \(\xi^+_i, i = 1, 2, ..., m\) in descending order and select the alternative with the highest \(\xi^+_i\).

In the above algorithm, the weights of attributes are known exactly before the decision making. In reality, there are often cases that the decision maker can not give exact weight values, that is, the information about the weights is incomplete, the decision maker only knows their partial knowledge. Under this situation, one is only sure that the weight vector \(w = (w_1, w_2, ..., w_n)\) satisfies \(w_i \geq 0, i = 1, 2, ..., n\) and \(w_1 + w_2 + ... + w_n = 1\). Generally, the incomplete attribute weight information can be expressed as follows:

1) A weak ranking \(\{w_i \geq w_j\}, i \neq j;\)
2) A strict ranking; \(\{w_i \geq w_j > 0\}, i \neq j;\)
3) A ranking with multiples \(\{w_i \geq \alpha w_j\}, 0 \leq \alpha_i \leq 1, i \neq j;\)
4) An interval form \(\{\beta_i \leq w_i \leq \beta_j + e_j\}, 0 \leq \beta_i \leq \beta_j + e_j \leq 1;\)
5) A ranking of differences \(\{w_i - w_j \geq w_j - w_i\}, i \neq j \neq k \neq l;\)

For a specific decision problem, the criteria weights can be described by a subset of the above relationships, the corresponding weight set is denoted by \(H\). The principle of the GRA is that we should choose the alternative which has the largest degree of grey relation from the PIS and at the same time has the smallest degree of grey relation from NIS. This principle results in the following multiple objective optimization problem for determining the weight vector.

\[
\begin{align*}
\max \xi^+= \sum_{j=1}^{m} w_j \xi^+_{ij}, i = 1, 2, ..., m, \\
\min \xi^- = \sum_{j=1}^{m} w_j \xi^-_{ij}, i = 1, 2, ..., m, \\
\text{s.t.} \quad w_1 + w_2 + ... + w_n = 1, \\
\quad w \in H, \\
\quad w_i \geq 0, i = 1, 2, ..., n.
\end{align*}
\]

Since the importance of different objectives is equal, we can transfer the above problem into the following single objective optimization problem.

\[
\begin{align*}
\max \xi = \sum_{j=1}^{m} \sum_{i=1}^{n} w_i (\xi^+_{ij} - \xi^-_{ij}), i = 1, 2, ..., m, \\
\text{s.t.} \quad w_1 + w_2 + ... + w_n = 1, \\
\quad w \in H, \\
\quad w_i \geq 0, i = 1, 2, ..., n.
\end{align*}
\]

Once the weight vector is determined, the algorithm can be used to rank the alternatives and the best one can be chosen.

IV. A REAL EXAMPLE ANALYSIS

There is a panel with the following five possible alternatives to invest money: \(A_1\) (car company), \(A_2\) (food company), \(A_3\) (computer company), \(A_4\) (arms company), \(A_5\) (a TV company). The following four attributes are considered: \(G_1\) (the risk analysis), \(G_2\) (the growth analysis), \(G_3\) (the social-political impact analysis), \(G_4\) (the environmental impact analysis). In this paper, the identification coefficient is \(\rho = 0.5\). The decision maker give the evaluation of the alternatives with respect to the attributes in the following decision matrix.
Step 1. Determine the PIS and NIS, respectively.

\[
\tilde{F}^+ = (([0.6,0.7],[0.1,0.2]),([0.6,0.7],[0.1,0.2])),
([0.7,0.7],[0.1,0.2]),([0.6,0.7],[0.1,0.2])),
\]

\[
\tilde{F}^- = (([0.3,0.4],[0.4,0.5]),([0.4,0.5],[0.3,0.4])),
([0.3,0.5],[0.3,0.4]),([0.3,0.4],[0.4,0.5])).
\]

Step 2. Calculate the grey relational coefficient of each evaluation value from PIS and NIS

\[
\xi = (\xi)_{ij} = \max(0.4286, 0.5750, 0.4286, 0.7500, 0.7000, 0.4286, 0.6000, 0.4286)
\]

\[
= (\xi)_{ij} = \max(0.3333, 0.4286, 1.0000, 0.7500, 0.7500, 0.6667, 0.6667, 0.3529)
\]

\[
= (\xi)_{ij} = \max(0.5789, 0.4783, 0.6471, 0.5348, 0.5789, 0.5789, 0.6471, 0.5348)
\]

\[
= (\xi)_{ij} = \max(0.3548, 0.5238, 0.4074, 0.8462, 0.5789, 0.5789, 1.0000, 1.0000)
\]

\[
= (\xi)_{ij} = \max(0.5354, 0.5336, 0.5212, 0.5612, 0.4954).
\]

Step 3. If the attribute weight vector is known exactly as \(w = (0.2,0.3,0.35,0.15)\), the degree of grey relational coefficient of each alternative from PIS and NIS can be computed by

\[
\xi^+ = 0.5732, \xi^- = 0.5900, \xi^+ = 0.6000, \xi^- = 0.6363,
\]

\[
\xi^+ = 0.6577, \xi^- = 0.5390, \xi^+ = 0.5157, \xi^- = 0.5513,
\]

\[
\xi^+ = 0.4976, \xi^- = 0.6699.
\]

Step 4. Calculate the relative relational degree of each alternative as

\[
\xi^+ = 0.5154, \xi^- = 0.5336, \xi^+ = 0.5212, \xi^- = 0.5612, \xi^+ = 0.4954.
\]

Step 5. Rank the alternatives according to the relative relational degree: \(A_i \succ A_j \succ A_k \succ A_l \succ A_m\), and thus the most desirable alternative is \(A_i\).

Now we consider the situation that the criteria weights are not known exactly. In order to determine the weight vector, we need to solve a linear programming problem. For instance, suppose that we only know that weights satisfy

\[
0.15 \leq w_i \leq 0.2, 0.2 \leq w_j \leq 0.3, 0.3 \leq w_i \leq 0.35,
\]

\[
0.1 \leq w_i \leq 0.3, w_i \leq 2w_j, w_i + w_j + w_i + w_i = 1,
\]

\[
w_j \geq 0, i = 1,2,3,4.
\]

The first two Steps are the same as the above method. In the Step 3, we set up the following linear programming to determine the weight vector.

\[
\begin{align*}
\max \xi &= -0.1769w_1 + 0.5787w_2 + 0.2705w_3 + 0.3394w_4, \\
\text{s.t.} & w_1 + w_2 + w_3 + w_4 = 1, \\
& w_2 \leq 2w_1, \\
& 0.1 \leq w_1 \leq 0.2, \\
& 0.15 \leq w_2 \leq 0.35, \\
& 0.15 \leq w_3 \leq 0.3, \\
& 0.2 \leq w_4 \leq 0.4.
\end{align*}
\]

By using the simplex method, we can get the optimal solution of the above problem as \(w_1 = 0.2, w_2 = 0.15, w_3 = 0.3, w_4 = 0.35\). Then the degree of grey relational coefficient of each alternative from PIS and NIS can be computed as follow:

\[
\tilde{c}_{ij}^+ = 0.5893, \tilde{c}_{ij}^- = 0.5886, \tilde{c}_{ij}^+ = 0.5143, \tilde{c}_{ij}^- = 0.5735,
\]

\[
\tilde{c}_{ij}^+ = 0.6934, \tilde{c}_{ij}^- = 0.5058, \tilde{c}_{ij}^+ = 0.4862, \tilde{c}_{ij}^- = 0.5736,
\]

\[
\tilde{c}_{ij}^+ = 0.5679, \tilde{c}_{ij}^- = 0.5742.
\]

Step 4. Calculate the relative relational degree of each alternative as

\[
\tilde{c}_{ij}^+ = 0.5381, \tilde{c}_{ij}^- = 0.5479, \tilde{c}_{ij}^+ = 0.4727, \tilde{c}_{ij}^- = 0.5025, \tilde{c}_{ij}^- = 0.5470.
\]

Step 5. Rank the alternatives according to the relative relational degree: \(A_i \succ A_j \succ A_k \succ A_l \succ A_m\) and thus the most desirable alternative is \(A_i\).

V. CONCLUSION

IVIFS is a useful tool to deal with problems with fuzziness and uncertainty, which often happen in real life. In this paper, a new multiple attribute decision making problem is studied based on IVIFS by using the GRA. We can solve the problem in two cases that attribute weight is known exactly and attribute weight is known partly. In the latter case, a linear programming is needed to solve to get the attribute weight. Finally, a numerical example is given to illustrate the method is feasible and effective.

REFERENCES


