Combined PAPR Reduction and Frequency Offset Estimation using Precoded Zadoff-Chu OFDM in WLAN System

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Abstract— The main objective of this paper is to implement a precoded Zadoff-Chu OFDM to reduce the Peak to Average Power Ratio (PAPR) and for frequency offset estimation in Wireless Local Area Networks. High PAPR and Carrier Frequency Offset are major disadvantages to the Orthogonal Frequency Division Multiplexing (OFDM) systems and can significantly degrade the power efficiency at the transmitter and performance of the system respectively. From the simulation results in MATLAB, it can be proved that using this method a PAPR of 2.5dB less than that of PTS technique can be achieved and the range of frequency estimation of N/2 subcarrier spacing can be obtained.

Keywords- Orthogonal Frequency Division Multiplexing (OFDM); Peak to Average Power Ratio (PAPR); Zadoff-Chu (ZC); Wireless Local Area Network (WLAN); Carrier Frequency Offset (CFO); Partial Transmit Sequence (PTS); Adjacent Symbol Combining (ASC).

I. INTRODUCTION

For wireless applications, an OFDM-based system is of interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation is realized using the Fast Fourier Transform (FFT) techniques[1]. However, such a transmission technique has major drawbacks like high PAPR and CFO. Peak power is caused by the large envelope fluctuations of the time-domain signal. High PAPR values lead to serious problems such as severe power penalty at the transmitter, which is not affordable in portable wireless systems where terminals are powered by battery. CFO is caused due to oscillator instabilities and Doppler effects.

Several PAPR reduction techniques have been proposed in the literature including Amplitude Clipping (AC), Block coding, Tone Reservation (TR), and Multiple Signal Representation (MSR) techniques such as the SeLective Mapping (SLM) and Partial Transmit Sequence (PTS). The simplest of PAPR reduction is the AC technique, that causes both in-band and out-of-band distortion. On the other hand, the Coding technique can offer excellent performance on PAPR reduction but the cost in its complexity and data rate loss has made it unpopular. The TR technique is popular in wired systems due to its low computational complexity, but the increase in the transmit signal power and associated degradation in bandwidth efficiency makes it undesirable in wireless systems. The SLM technique, can achieve excellent PAPR reduction with a high signal processing complexity due to the use of multiple Inverse Fast Fourier Transform (IFFT) operations per OFDM block. Similar to the SLM technique, the PTS technique also requires several IFFT operations per OFDM symbol but produces better PAPR performance than the SLM technique. It also has a higher complexity requirement and requires more side information bits. Both the SLM and PTS techniques are of immense interest to many researchers who have proposed modifications with the aim to reduce the complexity and improve the performance of these techniques [2]-[5]. The CCDF of the Precoded Zadoff-Chu OFDM is compared with the well known methods for PAPR reduction like PTS [6] and ASC [7] and it is found to provide 4.5dB reduction.

In the preamble-based carrier synchronization methods, the format of the preamble and the estimation algorithm applied are important to determining the carrier offset in the burst. In most WLANs standards, the preamble used to estimate the frequency offset consists of the repeated symbols. The synchronization algorithm is based on finding the highest correlation between two repeated sample sequences. Minn analyzed what reasons lead to a large variance in the Schmidl method, and proposed a novel structure of the preamble and defined a new timing metric to estimate the timing offset. The range of frequency offset estimation is enlarged to ±4 subcarrier spacing when compared to Schmidl which is ±1 subcarrier spacing. Morelli designed a new preamble with more short identical parts and proposed an improved frequency offset estimator. Song proposed a multistage approach to estimate the frequency offset with a large range based on the similar structure preamble. But the more short repeated identical parts in the preamble will cause a larger variance in timing offset estimation. In frequency synchronization, Moose work is based on frequency domain estimation [8] and Beek’s method on cyclic prefix [9]. The estimation range for these two methods is ±0.5 subcarrier spacing. The other methods like Schmidl [10], Minn [11], Ren [12], Morelli and Mengali [13], Multistage [14], Envelope Equalized Processing (EEP) [15] taken for comparison are based on time domain estimation techniques for frequency offset. The structure of the preamble differs in these various methods as a result the frequency offset estimation ranges also varies from ±1, ±2, ±4, ±8, ±16 and ±32 subcarrier spacing respectively in OFDM based WLAN system with 64 subcarriers.
The primary objective of this paper is to provide an algorithm for both PAPR and frequency offset estimation for OFDM based WLAN system. The importance of frequency offset estimation is found in Wireless Multimedia Communication Systems (Kim 2009), cellular systems (Kim 2010), cooperative OFDM System in Wireless Digital Broadcasting (Lee 2009).

The analysis followed here is based on Data-Aided (DA) synchronization algorithms that depend on the signal being transmitted. If the transmitted data is known at the receiver, the synchronizer is said to be Data-Aided.

In this paper, the precoded zadoff chu sequence is used for PAPR reduction. This technique is simple to implement and it also is signal independent. Further using the same sequence, frequency offset estimation is obtained. The range of frequency offset estimation obtained is N/2 subcarrier spacing.

In the Section II and III, the definitions of PAPR and CCDF is given. Section IV and V gives a description of PTS and ASC technique respectively. The proposed precoded Zadoff-Chu OFDM is explained in Section VI. Simulation results are given in Section VII. Conclusions are given in Section VIII.

II. THE PAPR OF A MULTICARRIER SIGNAL

A multicarrier signal is the sum of many independent signals modulated onto sub channels of equal bandwidth. The collection of all data symbols is denoted as Xn, n = 0, 1, ..., N – 1, as a vector X = [X0, X1, ..., XN-1] is termed a data block. The complex base band representation of a multicarrier signal consisting of N sub carriers is given by (1).

\[ x(t) = \frac{1}{N} \sum_{n=0}^{N-1} X_n e^{j2\pi n f \Delta t} \quad 0 \leq t \leq NT \]  

where \( j = \sqrt{-1}, \Delta f \) is the sub carrier spacing, and NT denotes the data block period. In OFDM the sub carriers are chosen to be orthogonal (i.e., \( \Delta f = 1/NT \)). The PAPR of the transmit signal is defined in (2).

\[ \text{PAPR}(x(t)) = \max_{0 \leq t \leq T} \left\{ \frac{1}{T} \int_{0}^{T} x(t)^2 \, dt \right\} \]

where \( x(t) \) represents the OFDM symbol in time domain, and \( T \) is the symbol duration. If the power of each subcarrier is normalized to 1 Watts, \( P_{\text{avg}} \) is N Watts. So the PAPR of uncoded BPSK OFDM systems with a frame size of N is equal to 10 \( \log_{10}(N) \) (dB).

III. CCDF

The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The complementary CDF (CCDF) of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. The CCDF of the PAPR of a data block is given as in (3). Statistics of the PAPR of an OFDM signal can be given in terms of its clipping probability or its complementary cumulative distribution function (CCDF).

\[ \text{CCDF}(\text{PAPR}(x(k))) = P(\text{PAPR}(x(k)) > x') = 1 - (1 - e^{-x'})^N \]

where \( N \) represents the time domain signal samples which are mutually independent and uncorrelated. where \( x' \) is the clipping level. This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some clip level \( x' \).

IV. PARTIAL TRANSMIT SEQUENCE

An input data block of N symbols is partitioned into disjoint subblocks in the PTS technique. The subcarriers in each subblock are weighted by a phase factor for that subblock. The phase factors are selected such that the PAPR of the combined signal is minimized.

In this technique, input data block \( X \) is partitioned into M disjoint subblocks \( X_m = [X_{m,0}, X_{m,1}, ..., X_{m,N-1}] \), where, \( m = 1, 2, \ldots, M \), and the subblocks are combined to minimize the PAPR in the time domain. The L-times oversampled time domain signal of \( X_m \) where, \( m = 1, 2, \ldots, M \), is denoted by \( x_m = [X_{m,0}, X_{m,1}, ..., X_{m,NL-1}]^T \), \( X_m \), \( m = 1, 2, \ldots, M \), is obtained by taking an IDFT of length NL on \( x_m \) concatenated with \((L - 1)N \) zeros. These are called Partial Transmit Sequences. Complex phase factors, \( b_m = e^{j\phi_m} \), where, \( m = 1, 2, \ldots, M \) are introduced to combine the PTSs. The set of phase factors is denoted as a vector \( b = [b_1, b_2, \ldots, b_M]^T \). The objective is to find the set of phase factors that minimize the PAPR. In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is written as \( P = \{ e^{j\phi_k}; \phi_k = 0, 1, \ldots, W - 1 \} \), where, \( W \) is the number of allowed phase factors. In addition, \( b_{\text{ref}} = 1 \) is set without any loss of performance[6].

So, an exhaustive search for \((M - 1)\) phase factors is performed. Hence, WM–1 sets of phase factors are searched to find the optimal set of phase factors. The search complexity increases exponentially with the number of subblocks M. PTS needs M IDFT operations for each data block, and the number of required side information bits is \( \log_2 W M - 1 \). The amount of PAPR reduction depends on the number of subblocks M and the number of allowed phase factors W. Another factor that affects the PAPR reduction performance in PTS is the subblock partitioning. This is the method of division of the subcarriers into multiple disjoint subblocks. The PTS technique works with an arbitrary number of subcarriers and any modulation scheme.

V. ADJACENT SYMBOL COMBINING

Similar to the PTS technique, the ASC technique is also based on a probabilistic approach that generates different representations for each OFDM symbol and transmits the one with the least PAPR. Unlike the PTS and technique, the ASC technique requires only one IFFT operation per OFDM block. To generate different representations for each OFDM symbol, the ASC technique exploits the variations between
different time-domain OFDM symbols. This is achieved by linearly combining two different time-domain symbols together using various mathematical operations, such as addition, subtraction, and complex-conjugate. The ASC mode works on two adjacent time-domain OFDM symbols and their complex conjugates. To clarify the operation of this approach, for two time-domain OFDM symbols, each parent set has four members (symbols or combination of symbols) as shown in Table 1. Any two members, that have the lowest PAPR, which are separable at the receiver, are selected for transmission. The parent sets consisting of \([x(1) \times x(2)]^T\) and \([x(1)^* \times x(2)]\) are not taken into consideration because their members have the same PAPR as those in \([x(1) \times x(2)]\) and \([x(1)^* \times x(2)]^T\) respectively. Similarly, not all possible members need to be included in the parent sets when they have the same PAPR as other members are already present in the parent set\([7]\).

**TABLE I. PARENT SETS AND MEMBER COMBINATIONS**

| P(1)=|[x(1) x(2)] | P(2)=|[x(1) x(2)^*] |
|---|---|
| x(1) | x(1) |
| x(2) | x(2)^* |
| (1/2)(x(1)+x(2)) | (1/2)(x(1)+x(2)^*) |
| (1/2)(x(1)-x(2)) | (1/2)(x(1)-x(2)^*) |

**VI. PROPOSED PRECODED ZADOFF-CHU OFDM**

High Peak to Average Power Ratio (PAPR) is caused by the large envelope fluctuations of the time-domain OFDM signal. High PAPR values lead to serious problems such as severe power penalty at the transmitter, which is not affordable in portable wireless systems where terminals are powered by battery.

The Carrier frequency synchronization is the maintaining of the same carrier frequency between the Local Oscillators at the Transmitter and the Receiver ends. The carrier frequency synchronization is utilized to eliminate the carrier frequency offset caused by the mismatch of the local oscillators between the transmitter and the receiver, nonlinear characteristic of the wireless channel as well as the Doppler shift. The receiver needs to find the correct Carrier frequency to avoid the ICI and degrade the Signal to Noise Ratio (SNR) performances. The degradation is caused by two phenomena,

- Reduction of amplitude of the desired sub carrier
- ICI caused by neighboring carriers

The amplitude loss occurs because the desired sub carrier is no longer sampled at the peak of the sinc function of the DFT. Adjacent carriers cause interference, because they are not sampled at the zero crossings of the sinc functions.

Zadoff-Chu sequences possess good correlation properties which are essential in a variety of engineering applications such as establishing synchronization between a mobile terminal and a base station, performing channel estimation, and reducing peak-to-average power ratio.

**A. Sequence design**

Polyphase sequences are finite, complex, time-discrete sequences with constant magnitude and variable phases. In mathematics, a polyphase sequence is a sequence whose terms are complex roots of unity as in (4).

\[ a_n = e^{2\pi j n / NG} \]  

(4)

where \(x_n\) is an integer. Polyphase sequence is not new in literature, infact it is also proposed to be used for IEEE 802.11n, IEEE 802.16m and 3GPP-LTE. Polyphase preambles (Frank and Chu sequences) belonging to the class of Constant Amplitude Zero Auto-Correlation (CAZAC) sequences are adopted in IEEE 802.16 and IEEE 802.15.3, due to their perfect autocorrelation property.

A pulse is referred to as a chirp pulse of the order \(s\), if and only if, its instantaneous frequency \(f(t)\) increases and/or decreases linearly within the pulse, and the first time derivative of the instantaneous frequency (the angular acceleration) is a step function with the number of time intervals where it is constant is equal to \(s\). The generalised chirp-like (GCL) sequences are complex, chirp-like polyphase sequences with perfect periodic autocorrelation and optimum cross-correlation properties.

**B. Sequence properties**

The time domain waveforms of the GCL-modulated OFDM signals have low PAPR. In addition, because of the use of different “classes” of GCL sequences, any pair of the sequences will have low cross correlation at all time lags, which greatly improves the code detection and Channel Impulse Response (CIR) estimation. The GCL sequence has the following important properties:

Property 1: The GCL sequence has constant amplitude, and its NG -point DFT has also constant amplitude.

Property 2: The GCL sequences of any length have an “ideal” cyclic autocorrelation (i.e., the correlation with the circularly shifted version of itself is a delta function)

Property 3: The absolute value of the cyclic cross-correlation function between any two GCL sequences is constant and equal to 1 / \(NG\), when \([u_1, u_2, u_1, u_2]\) are all relatively prime to \(NG\) (a condition that can be easily guaranteed if \(NG\) is a prime number). Compared with BPSK or even QPSK preambles, the complex-valued GCL sequences can be systematically constructed with guaranteed good PAPR and good correlation.

The Frank-Zadoff-Chu sequences are a sub-class of the GCL family of spreading sequences. Zadoff-Chu has the property of sharp auto-correlation value and zero side lobes. In addition, Zadoff-Chu sequence is also Zadoff-Chu sequence after FFT or IFFT.

**C. Precoding and CFO estimation**

Zadoff-Chu sequences are a class of unity-modulus polyphase sequences with impulse-like periodic AutoCorrelation (AC) functions. The Zadoff-Chu sequences are a class of polyphase sequences of length \(N\) and the elements are given by (5).
\[ a_u(n) = e^{-j2\pi un(n+1)/N} \]  
(5)

where \( n = 0, 1, 2, \ldots, N-1 \) and \( u = 1 \). The ZC sequence is generated and multiplied with the data to be transmitted.

The proposed OFDM systems can be easily implemented by Inverse Fast Fourier Transform (IFFT). After precoding the \( N \) randomly generated symbols, the OFDM baseband signal is obtained by applying the IFFT. Thus the modulated OFDM signal with \( N \) subcarrier can be expressed as in (6).

\[ x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P_k e^{j2\pi nk/N} \]  
(6)

where \( P \) is the precoder while \( x_n \) and \( X_k \) are the time-domain and frequency domain signals respectively. For Zadoff-Chu based precoder, \( P \) is given by \( a_u(n) \). The PAPR of this precoded Zadoff-Chu OFDM signal is simulated in MATLAB.

The received signal is represented as \( r(k) \) in (7), \( \varepsilon \) represents the frequency offset, \( x(k) \) is the transmitted signal and \( n(k) \) represents additive white gaussian noise.

\[ r(k) = x(k)e^{-j2\pi nk/N} + n(k), \quad k = 0, 1, \ldots, N-1 \]  
(7)

To the receiver, the synchronization sequence in the transmitter should be known. Here in this proposed frequency synchronization algorithm, the synchronization sequence is the Zadoff-Chu sequences \( a_u(k) \) instead of original preamble to accomplish frequency offset estimation. In the process of synchronization, Zadoff-Chu sequence must do conjugate relation with the received signal first as in (8) to eliminate the influence of synchronous sequence.

\[ y(k) = r(k) a^*_u(k) \]  
(8)

The length of Zadoff-Chu sequences is \( L \). If \( N \) is the number of subcarriers used, then \( N/L \) number of such sequence will be present in the OFDM symbol. The second step in frequency offset estimation is to find the Correlation Function. The correlation function \( \theta \) is calculated as in (9).

\[ \theta = \sum_{i=0}^{2L-1} \sum_{k=0}^{L-1} y(k+iL)(y(k+(i+1)L)) \]  
(9)

Finally, the frequency offset estimation is obtained as in (10).

\[ \varepsilon = -\text{arg} (\theta) \frac{N}{nL} \]  
(10)

VII. SIMULATION RESULTS

The CCDF of Conventional OFDM, Zadoff-Chu-based OFDM, PTS technique-based OFDM, and ASC technique-based OFDM for \( N = 64 \) subcarriers based WLAN system is simulated in MATLAB as shown in Figure 1.

VIII. CONCLUSION

The proposed precoded Zadoff-Chu sequence OFDM achieves a PAPR of approximately 2.5dB less than PTS technique in OFDM based WLAN system. From the simulated results in MATLAB, it can be concluded that precoded Zadoff-Chu OFDM achieves both PAPR reduction and frequency synchronization in OFDM based WLAN systems.
REFERENCES


