A Improved Multi-objective Evolutionary Algorithm Based on Three-way Radix Quicksort

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Abstract—A improved multi-objective evolutionary algorithm based on Three-way radix quicksort (TQIEA) is presented in this paper for multi-objective optimization problems (MOPs). This algorithm uses the idea of three-way radix quicksort to divided the population into three sections, Recursively sort the different sections until all the individuals have been classified and assigned the fitness value. The proposed algorithm is validated by 6 benchmark test problems. Compared with four other state-of-the-art multi-objective algorithms, TQIEA achieves competitive results in two quality indicators.

Keywords: evolutionary algorithm, multi-objective optimization, Three-way radix quicksort

I INTRODUCTION

Multi-objective Evolutionary algorithms (MOEA), investigated by many researchers is well suited for tackling multi-objective optimization problems (MOPs) because of their exploration and exploitation ability to find multiple trade-off solutions in the search space. NSGAII, as a famous MOEA, was proposed by Deb et al, it employed the nondominated sorted mechanism and assigns the fitness value by the amount of the nondominated individuals dominated by the specified individuals, in the evolution, the best solutions were select to construct the nondominated solutions set. However, most solutions tend to lie at the current trade-off front in the evolution, there was no distinct selection advantage for most of these solutions, so this methods took a expensive computational and it was difficult to find the better solutions. How to select the the better solutions is the key of the algorithm, without an adequate selection methods, it is difficult to find better solutions in the search space, especially for the many-objective problems and some complicated multi-objective problems. Therefore, it is worthwhile work to devise an effective selection method to handle these complicated problems.

Jensen MT pointed out that selected the nondominated individuals and assigned the fitness value for the individuals had occupied most of the run time for the MOEAs, and that the main reason for the low efficiency of the MOEAs; SHI Chuan, et all found that there were many comparisons among the individuals should be deleted, and indicated that some comparisons could be derived by the relationships that existed. Jinhua Zheng proved that the individuals of an evolutionary population could be sorted by quick sort.

Motivated by these papers and based on three-way radix quicksort, a quick selection methods is devised to select the nondominated individuals from the population, the aim is to improve the efficiency of the algorithm by reducing the comparisons of the dominated relationships among the individuals.

The remainder of this paper is organized as follows: In section 2, TQIEA, our proposal for solving MOPs, is described; thereafter, in section 3, several test problems are used to evaluate TQIEA’s effectivity by comparing with NSGAII, SPEA2, PESA-II and MOCell; Finally the main conclusion and suggest some of the further research is presented.

II THE QUICK SELECT ALGORITHM BASED ON THREE-WAY RADIX QUICKSORT

A. the Relationship of Individuals in Population

Before discussing how to use three-way radix quicksort to select the nondominated solutions, the relationships among the individuals are analyzed: there are many relationships could be derived by the existed relationships, and some of the comparisons are superfluous. As the feature of the Pareto dominating relationship of individuals have the transitivity, for example $N_i \succ N_j$, $N_j \succ N_k$ then $N_i \succ N_k$ , the comparison between $N_i$ and $N_k$ could be deleted, and all the individuals is dominated by $N_i$ should be dominated by $N_k$. On the other hand, if $N_i \succ N_j$, $N_j \succ N_k$, $N_k \succ N_i$ , then the comparisons of $N_i \& N_j$, $N_j \& N_k$, $N_k \& N_i$ are unnecessary when it
only need to select the best solutions. So in the selection, how to delete this redundant relationships is the key of efficiency.

B. Three-way Radix Quicksort

Quicksort is one of the fastest and simplest sorting algorithms. It works recursively by a divide-and-conquer strategy[9]. Based on the quicksort, Bentley and Sedgewick[10] proposed three radix quicksort to sort the strings, the Principle of three radix quicksort is to pick an element from the pivot of an array. Consider the first character (key) of the string (multikey). Partition the remaining elements into three sets: those whose corresponding character are less than, equal to, and greater than the pivot's character. Recursively sort the "less than" and "greater than" partitions on the same character. Recursively sort the "equal to" partition by the next character (key), the details see Ref 10.

C. the Quick Select algorithm based Three-way Radix Quicksort

Based on the idea of three-way radix quicksort and the relationship of the individuals in the population, an improved quick select algorithm is devised to constructed the nondominated solutions sets in the evolution. For the individuals in the population Pt, there are two steps:

**step 1:** A radix individual xi is selected from the population randomly, the individuals in the population Pt are compared with xi, then the Pt is divided into three sections. Pfp, Pmp, Plp. Pfp is the front section that the individuals are dominated by xi, Pmp the middle section that the individuals are dominated by xi, Plp the later section that the individuals dominated xi, the fitness of individuals in Pmp, Plp is calculate as:

\[ f(x_i) = \min \{x_j \mid x_j \in P_t \land x_j \succ x_i\} \quad (1) \]

\[ f(x_i) = \max \{x_j \mid x_j \in P_t \land x_j \preceq x_i\} \quad (2) \]

\[ f(x_i) = \max \{x_j \mid x_j \in P_t \land x_j \preceq x_i\} \quad (3) \]

**Step 2:** Select a radix individual xj to compared with the individuals in Pfp, Pmp, Plp respectively, the fitness of individuals in Pfp is calculate as:

\[ f(x_j) = \min \{x_i \mid x_i \in P_{fp} \land x_i \succ x_j\} \quad (4) \]

the fitness of individuals in Pmp, Plp is calculate as:

\[ f(x_j) = \min \{x_i \mid x_i \in P_{mp} \land x_i \preceq x_j\} \quad (5) \]

the fitness of individuals in Pmp, Plp is calculate as:

\[ f(x_j) = \min \{x_i \mid x_i \in P_{mp} \land x_i \preceq x_j\} \quad (6) \]

This quick select algorithm reduce comparison with the dominated individuals, for the example mentioned in above, N1, N2, N3, N4, N5, N6, N7 are arranged in the front of N8, so their comparison are ignored in the next nondominated sort. At the same time, the feedback is introduced to improve the efficiency of the selection.

III Experiment

This section is devoted to the evaluation of the TQIEA. First, the test problems are selected from the literature. Then, the quality indicators which have been used to measure the equality of the algorithm are described. At last a comparison against four famous MOEAs are performed.

A. Test Instance

A numbers of well-know benchmark multi-objective test problems are selected from the standard literature on EMO such as Schaffer, Kursawe, as well as some diverse complexities problems like the ZDT1 and ZDT3 problems (which were developed by Zitzler), and the DTLZ1 and DTLZ2 problems (which were defined by Deb et all), (noted :for the DTLZ1 and DTLZ2, the amount of objectives function are three).

B. Quality Indicator

Two quality indicators are used to measure the convergence behavior and diversity of solutions. Generation Distance[12] is applied to measure the convergence performance of the algorithm, Spread(Δ)[13] which was introduced by Deb is used to measure the diversity of the solutions set. Their Pareto Front of this benchmark problems have been obtained by using enumeration search strategy[13].

C. Experimental result

The results of the quality indicators are shown by box-plots[14]: the statistical values of GD and Δ for the test samples obtained by TQIEA, NSGAII, SPEA2, PESAII and MOCell are shown in fig 1.a and fig 1.b. Each number in figures correspond to the algorithm are shown Table I

<table>
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<tr>
<td>Algorithm</td>
<td>TQIEA</td>
<td>NSGA-II</td>
<td>SPEA2</td>
<td>PESA-II</td>
<td>MOCell</td>
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In terms of GD, TQIEA obtained the best performance in Schaffer, ZDT1, DTLZ1 and DTLZ2 while MoCell,
demonstrated comparative good measures of Kur and

![Box plots of GD values for different algorithms](image)

Figure 1. (a) value of GD

Fig1 statistical value of GD and spread for SCH, Kur and ZDT1, ZDT3, DTLZ1, DTLZ2 obtained by TQIEA, NSGAI, SPEA2, PESA-II and MoCell. The distributions of these samples have been illustrated by the box plots, in a notched-box, a robust estimate of the uncertainty about the medians for box-to-box comparison could be represented by the notches. symbol + denote outliers

ZDT3. With regard to $\Delta$, it seems that TQIEA did better than other four algorithms in this experiment. With the aim of giving a complete graphical overview of the behavior of TQIEA, we simulated the Pareto fronts for Kur, Schaffer, ZDT1 in Fig. 2,3,4 obtained by different algorithms, we can observe that for the problem of

![Pareto fronts of different algorithms](image)

Kursawe, observe that for the problem of Kursawe, the TQIEA and MoCell obtained the better spread than the other three algorithms; for the problem of Schaffer, the TQIEA outperformed than the other algorithm, for the problem of ZDT1, TQIEA,MoCell and SPEA2 performed better than NSGA-II and PESA-II.

![Pareto fronts of different algorithms](image)

Figure 2. TQIEA finds a better spread of solutions than SPEA2, NSGA-II, PESA-II in the Kursawe problem
Overall, considering the results of the experiments, it obviously that TQIEA is an efficient algorithm in solving MOPs because TQIEA obtained the competitive values in test problems and it performed very stable in terms of convergence, diversity.

IV CONCLUSION

In the field of multi-objective optimization algorithm, how to improve the efficiency of selecting nondominated solutions and preserving the diversity of solutions is the key of the algorithm. Therefore, based on three-way radix quicksort, TQIEA is proposed to select the nondominated solutions in the population based on the concept of Pareto domination. Two quality indicators are used to compare the performance of TQIEA with the other famous multi-objective optimization algorithms. The results of the quality indicator indicate that TQIEA is a competitive and effective method considering the convergence and diversity measures.

A matter of future work is the application on solving the real-world problems. In this sense, we intend to employ TQIEA to solve complicated problems in the multi-shop scheduling with mixed messages.

ACKNOWLEDGEMENT

This work is support by the Natural Science Foundation of China (grant no: 50775089), the National High Technology Research and Development Program(Grant No: 863 Program) of China (Grant no:2007AA04Z190 and 2009AA043301), the National Basic Research Program (973 Program) of China (Grant no: 2005CB724100 ), and the Natural Science Foundation of
Guangdong Province, P. R. China (Grant No: 8351503101000001).

REFERENCE


