Application of Adaptive Filtering Algorithm Based on Wavelet Transformation in Aeromagnetic Survey

Deng Peng, Lin Chun-sheng, Tan Bin , Zhang Jian
Department of Weaponry Engineering
University of Naval Engineering
Wuhan, China
hbgal@126.com, LCS_and_zh@163.com, yctanbin@163.com, edward5433@163.com

Abstract—Traditional method of compensating aircraft magnetic inference is complicated. In order to solve coefficients of inference, aircraft must have a box wide sides study flight before survey. However due to multi-collinearity of the traditional aircraft’s magnetic inference model, it is difficult to estimate the coefficient of model accurately, and the aircraft’s inference couldn’t be removed clearly. The adaptive filtering algorithm based on wavelet transformation uses result of wavelet transformation for measured magnetic signal to be the reference signal of adaptive filtering. The target signal isn’t correlated with noise, so the filter effect works well and the aircraft’s magnetic inference compensated well. The method could simplify the process of compensation and reduce the learning time in the air. By using this method to measured data, aeromagnetic survey could detect magnetic anomaly signal both aircraft in violent shaking and smooth flight without using any another equipment such as GPS.

Keywords—aeromagnetic survey; wavelet transformation; adaptive filter; magnetic anomaly detection

I. INTRODUCTION

The magnetic inference of the aircraft is consisted with permanent source interference field, induced source interference field and eddy-current interference. These inferences fluctuate as the pilot varies any aircraft control, such as changing the engine speed, adjusting ailerons, etc. The statistical characteristics of magnetic interference changes too. In aeromagnetic survey, the interference caused by aircraft is larger than the ma.

Traditional aircraft compensation model use 16 terms or 18 terms\(^1\) of linear equation to represent the permanent, induced and eddy-current interference due to an aircraft as a function of the attitude. Four flight paths are normally flown at high altitude in a box wide side parallel, and perpendicular paths to the proposed survey azimuths with a series of pitch, roll and yaw movements. The data from these flight paths are then used to determine coefficients relating to the motions, the linear equation system is then usually solved with least-squares method\(^2-4\). The resulting coefficients are then used to remove the aircraft’s magnetic effects from survey data flown at lower altitude. However the compensation model is a morbid model and there is badly multi-collinearity exists in it. It is difficult to reach an accurate model and the noise caused by aircraft could not be removed clearly.

In practical magnetic signal processing, when an aircraft flown in the air, it is difficult to measure the statistical characteristics of magnetic signal and noise and it is hard to use Wiener filter or Kalman filter for optimal filtering but adaptive filtering. However, in adaptive filtering, the acquisition of reference signal is a major problem. The best reference signal for the received signal should be independent of the target signal and associated with noise. Wavelet transform as a time-varying non-stationary signal analysis tools, is useful on time-frequency analysis\(^5-7\). Wavelet transform can decompose the received signal into different frequency bands to achieve the election of frequency. There are some methods using wavelet transform in de-noising, such as principle of modulus maximum de-noising, wavelet domain correlation coefficient de-noising and wavelet threshold de-noising. Although these methods can separate signal and noise, but because both of two overlap in the frequency domain, therefore these methods are not optimal filtering. Filtered noise is correlated with the noise in received signal and little correlated with target signal. For adaptive filtering applications which are difficult to obtain the ideal reference signal, the noise after wavelet transforming de-noising could be the best reference signal.

II. ADAPTIVE FILTERING

In 1950s, Windrow and Hoff proposed adaptive filtering algorithm based on minimum mean square (LMS). LMS algorithm has simple structure and the robustness is good. Besides it is easy to hardware realization and real-time processing. In particular, it is the first practical algorithm derived from the statistical analysis. Which is the basis for a class of adaptive filter, and widely used in various fields.

Adaptive filter based on minimum mean square error is through adjusting its own impulse response \(h(n)\) automatically to achieve the minimum mean square error filtering effect. The basic principle is that adjusting filter weight vector \(\mathbf{w}(n)\) to maintain the mean square error \(E[e^2(n)]\) minimal. So that the output signal \(y(n)\) gradually approach the reference signal \(d(n)\), while the estimation error \(e(n)\) between \(y(n)\) and \(d(n)\) adjust weight vector \(\mathbf{w}(n)\) to achieve the best filtering under the adaptive algorithm.

Adaptive filter is consisted with digital filter and adaptive algorithm. The parameters of filter are adjustable with the algorithm. Because the objects and function are different, so the structures of digital filters are different, such as FIR filter,
IIR filter, Lattice-type filter, et al. There are different adaptive algorithms based on different criteria, such as least mean square (LMS), the largest signal to noise ratio (MSNR), recursive least squares (RSL), et al.

The LMS algorithm is a stochastic gradient algorithm in that it iterates each tap weight of the transversal filter in the direction of the negative instantaneous gradient of the squared error signal with respect to the tap weight in question. The simplicity of the LMS algorithm coupled with its desired properties, has made it and its variants an important part of the adaptive technique. LMS recursion can be written as:

\[ x(n) = [x(n), x(n-1), \ldots, x(n-m+1)]^T \]

Weight vector:
\[ w(n) = [w_{m1}, w_{m2}, \ldots, w_{mn}]^T \]

Output of filter:
\[ y(n) = \sum_{i=1}^{m} w_m(n)x(n-i+1) = w^T(n)x(n) \]

Error between output of filter and desired signal:
\[ e(n) = d(n) - y(n) = d(n) - w^T(n)x(n) \]

According to the criteria of the minimum mean square error (MSE), the weight of best filter is adjusted to minimize the mean square error \( E[e^2(n)] \). The optimal of which can be found using recursive convergence.

\[ w(n+1) = w(n) + \mu e(n)x(n) \]  \( (1) \)

Where \( \mu \) is the step-size. Convergence condition is \( 0 < \mu \leq 1/\lambda_{\text{max}} \), \( \lambda_{\text{max}} \) is the maximum eigenvalue of input signal autocorrelation matrix.

III. DISCRETE WAVELET TRANSFORM

The wavelet transform is a time-frequency decomposition which uses a basis of functions known as wavelets. In the continuous wavelet transform (CWT) derivation, most of the information related to close scales or times is redundant. These procedures result in a high computation cost. Which in same cases, could be solved using an adequate discrete wavelet transform (DWT), which uses discrete values of scale \( j \) and localization \( k \). Define the DWT as:

\[ Wf(u) = \int_{-\infty}^{\infty} f(u)\psi_j^k(u)\,du \]  \( (2) \)

where
\[ \psi_j^k(u) = 2^{-j/2}\psi(2^{-j}u-k). \]

Such sets of wavelet functions are orthogonal and their respective functions are translated and dilated. Signals \( f(u) \) are represented by series such as

\[ f(u) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_j^k\psi_j^k(u) \]  \( (3) \)

where \( \psi_j^k(u) = \psi(2^j u - k) \) are wavelet function and \( d_j^k \) are wavelet coefficients

\[ d_j^k = \int f(u)\psi_j^k(u)\,du \]  \( (4) \)

As a property of the wavelet transform, it is possible to show that the amplitude of the wavelet coefficients is associated with abrupt signal variations or “details” of higher frequency\(^{[2]}\).

A mother-wavelet function is generated form a scaling function. It obeys the scale relation

\[ \phi(x) = 2\sum_k h(k)\phi(2x-k) \]  \( (5) \)

where \( \phi(x) \) is known as the scale function, and \( h(k) \) is a low pass-filter. Then, the mother-wavelet functions are built as

\[ \psi(x) = \sum_k g(k)\phi(2x-k) \]  \( (6) \)

where \( g(k) = (-1)^{k+1}h(1-k) \) is a high-pass band filter.

From this mother wavelet, it is possible to build up functions \( \psi_j^k \) that can be dilated and contracted. They are from a Riesz basis for the “detail” space. This represents the difference of information between one generation and the next generation, i.e.

According to the multi-scale decomposition of wavelet transforms, signals \( S \) is decomposed just like Fig. 1. \( D1, D2, \ldots, Dn \) are low-pass wavelet filters, and \( A1, A2, \ldots, An \) are high-pass wavelet filters. The “detail” of signals is usual in high frequency. \( D1, D2, \ldots, Dn \) contain the statistical characteristics of noise. They could used to be the reference signal in adaptive filtering.

IV. ADAPTIVE FILTER BASED ON WAVELET TRANSFORM

When the frequencies of target signal and noise are not overlap, it could use traditional FIR or IIR filter to filtering. When they are partly overlap, traditional wavelet transform could be used to de-noising. But the filtered signal is not the optimal estimation for the target signal. The noise decomposed by DWT is correlation with the original noise. That is the best reference signal to adaptive filter.
Figure 2. Structure of adaptive filter based on DWT

As Fig. 2 shown, input signal is \( x(n) = f(n) + n(n) \).

Where \( f(n) \) is target signal, and \( n(n) \) is noise.

\( d(n) \) is reference signal decomposed form input signal by DWT.

According to the LMS algorithm, the adaptive rule is

\[
E \left[ c^2(n) \right] = \text{min} \tag{7}
\]

Weight vector of filter adjusts following this rule and output signal approaching to target signal at the same time.

\[
w(n+1) = w(n) + 2\mu e(n) \tag{8}
\]

V. MAGNETIC DETECTION OF AEROMAGNETIC

The following 18 term aircraft magnetic interference model was presented by Leliak:

\[
H_c = c_1 \cos X + c_2 \cos Y + c_3 \cos Z \\
+ H_x \left\{ c_{10} \cos X \left( \cos X \right)' + c_{11} \cos X \left( \cos Y \right)' + c_{12} \cos X \left( \cos Z \right)' + c_{13} \cos Y \left( \cos X \right)' + c_{14} \cos Y \left( \cos Y \right)' + c_{15} \cos Y \left( \cos Z \right)' + c_{16} \cos Z \left( \cos X \right)' + c_{17} \cos Z \left( \cos Y \right)' + c_{18} \cos Z \left( \cos Z \right)' \right\} \\
= \sum_{i=1}^{18} c_i a_i \tag{9}
\]

Where \( H_c \) is the earth’s magnetic field, \(( \cos X, \cos Y, \cos Z )\) is the directional cosines of the earth’s field vector with the transverse, longitudinal and vertical axes of aircraft, \(('')\) is the differentiation operator with respect to fiducial, \( H_x \) is the total intensity of the interference field, \( c_i, 1 \leq i \leq 18 \), are the inference coefficients. Usually, \(( \cos X, \cos Y, \cos Z )\) are contained by using GPS orientation information. Once the coefficients are obtained we have an explicit model for predicting aircraft interference;

interference effects are eliminated by subtracting the calculated \( H_c \) from the magnetometer signal.

Note that the first 3 term represent the permanent source interference field, followed by 6 terms standing for the induced source interference field, and the last 9 terms express the eddy-current interference.

Take the magnetometer signal as an adaptive model. The noise is aircraft magnetic interference. It is fluctuate as any aircraft control. The target signal is magnetic anomaly signal. The reference signal decomposed from input signal by DWT.

Applying this adaptive filtering algorithm on the practical aeromagnetic survey. The magnetometer signal was measured by a Cesium magnetometer on a fixed wing aircraft. The data was collected every 0.05s or the sampling rate is 20Hz. The aircraft was in smooth flight, the speed is 100m/s, the attitude is 300m, and the flight direction is from truth south to truth north. Output form adaptive filter then passed through a 9th-order low-pass Butterworth filter.

The magnetometer signal of Fig. 3 is measured on one flight path. The maneuvers of the aircraft were pitch(\( \pm 5^\circ \)), roll(\( \pm 5^\circ \)) and yaw(\( \pm 10^\circ \)). The flight altitude is 800m and the flight direction is along SN direction. Comparison of three maneuvers, the magnetometer signal of pitch is fluctuates largest. Then the data of pitch is just as been an example of aircraft’s maneuver in aeromagnetic survey to be analyzed. The target signal is provided from [10]. The length of target signal is 200 points and the maximum point-to-point difference is 1.5nT. It is added on 1000-1200 of data of pitch.
Fig. 5 shown, after about 400 points adjusting, error curve of adaptive filter based on DWT was steady. Fig. 6 is shown the last 600 points of data. The top panel is input signal of pitch. The middle panel is output signal. It was passed through a 9th-order low-pass Butterworth after adaptive filter. At the beginning, the compensation of magnetic inference was not match very well, there is same fluctuate. When adaptive filter was steady, the output fluctuated was small and the maximum point-to-point difference was within 1nT. Between the 1000 and 1200 point, the output have detected a MAD signal, which is the target signal. The swing of detected MAD signal is more than 3nT, comparison of pitch of aircraft caused nearly 30nT fluctuate.

Only through DWT, the inference couldn’t be compensated and the MAD signal couldn’t be detected. As Fig. 7 is the data of same aircraft, which was flown smoothly without any maneuver. The direction is from north to south. The target is added on 1000-1200 points of data too, and the magnitude is a quarter of former. The maximum point-to-point difference is 0.375nT. Clearly, between the 1000 and 1200 point, a MAD signal was detected. So the detection area of MAD signal is larger than former.

VI. CONCLUSION

In order to compensate the magnetic inference caused by aircraft maneuver, traditional compensation is having a box wide sides study flight before survey. The adaptive filtering algorithm intergrated with wavelet transformation uses adaptive model as aircraft magnetic interference model, instead of 16 or 18 terms model. The data measured by magnetometer firstly through DWT, and the detail signal is adopted to be the reference signal of adaptive filter. Because the detail signal of DWT is correlated with the noise of measured signal and little correlated with target signal, so the de-noising effect is very well. Aeromagnetic survey adopted this method could detect MAD signal both aircraft in violent shaking and smooth flight without using any another equipment such as GPS. The process of study is simpler than traditional method and the study time is shorter.
REFERENCES


