Abstract—Airport gate assignment is to appoint a gate for the arrival or leave flight and to ensure that the flight is on schedule. Assigning the airport gate with high efficiency is a key task among the airport ground busywork. As the core of airport operation, aircraft gate assignment is known as a kind of complicated combinatorial optimization problem. This paper proposed robust assignment model to minimize the overall variance of slack time. According to the intrinsic characteristics of the objective function itself, a Tabu search algorithm and meta-heuristic method is given to solve the gate assignment problem and the numerical simulations are mode. The experimental results show that the meta-heuristic TS and random algorithm TS to compare the results, with improvement of 60.63%, thereby validate the meta-heuristic method and TS in the airport gate assignment problem of model application is feasible and efficient.

Keywords-Gate assignment; overall variance of slack time; meta-heuristic method; tabu search algorithm

I. INTRODUCTION

The primary purpose of flight-to-gate assignments in airports is to assign aircraft to gates to meet operational requirements while minimizing the overall variance of slack time periods. The term gate is used to designate not only the facility through which passengers pass to board or leave an aircraft but also the parking positions used for servicing a single aircraft. These station operations usually account for a smaller part of the overall cost of an airline’s operations than the flight operations themselves. However, they can have a major impact on the efficiency with which the flight schedules are maintained and on the level of passenger satisfaction with the service.

Various methods and optimization techniques have been devoted to study the Airport Gate Assignment Problem (AGAP) by many researchers in past years. For example, Braaksma and Shortreed [1] provided one of the first attempts to use quantitative means to minimize intra-terminal travel through the design of terminals. Babic et al. [2] formulated the gate assignment problem as a 0-1 integer program and used a branch-and-bound algorithm to find solutions where transfer passengers were not considered. Mangoubi and Mathaisel [3] used an LP relaxation of an integer program formulation with greedy heuristics to solve the problem of Babic et al. A. Bolat [4] proposed an optimal branch and bond procedure and a heuristic algorithm to solve the problem to minimize the range of slack times. Baron [5] used simulation to analyze the effects of passenger walking distance resulting from different gate-use strategies where both local and transfer passengers are considered. Simulation models for the problem can also be found in, for example, Cheng [6,7]. Some models have suggested ways to resolve stochastic flight delays in the planning stages. Yan and Chang [8] and Yan and Hou [9] added a fixed buffer time between flights assigned to a gate. The gate assignment with time windows is studied in [10]. Other research on the AGAP includes [11-13].

The remainder of the paper is organized as follows: Formal definition of the problem and the related mathematical model are given in Section 2. A tabu search algorithm and meta-heuristic implementation is proposed in Section 3. Section 4 contains an extensive set of computational experiments and comparisons. Finally, summary and conclusions appear in Section 5.

II. PROBLEM DESCRIPTION AND FORMULATION

In this section, we provide a model for the AGAP which attempts to assign flights to gates to minimize the overall variance of slack time. Before modeling the AGAP, the definitions of variables for AGAP are shown as follows:

- $a_i$: arrival time of flight $i$;
- $d_i$: departure time of flight $i$;
- $R_k$: the last slack time at gate $k$ until the arrival of flight $i$;
- $\bar{R}$: the mean of slack time in gates;
- $ST_k$: earliest available time of gate $k$ in the beginning of planning period;
- $ET_k$: latest available time of gate $k$ in the end of planning period;
- $e_k$: the type of gate $k$, if it is a large gate, its value is greater;
- $v_i$: the type of flight $i$, if it is a large flight, its value is greater;
- $I$: dwell time between two continue flights assigned to the same gate;
\( y_{ik} \) : the decision variable, \( y_{ik} = 1 \) if flight \( i \) is assigned to gate \( k \), else \( y_{ik} = 0 \);

\( z_{ijk} \) : the decision variable, \( z_{ijk} = 1 \) if flight \( i \) and flight \( j \) are both assigned to gate \( k \), else \( z_{ijk} = 0 \);

\( S \) : a large enough positive number.

A. Mathematical Model

Without loss of generality, it is assumed that the flights are sorted in ascending order of their arrival times. The following mixed-binary quadratic programming model is proposed to assign the flights to gates with the minimum overall variance of slack time:

Minimize \( f = \sum_{i=1}^{N+1} \sum_{k=1}^{M} (R_{ik} - \bar{R})^2 \)  \hspace{1cm} (1)

Subject to:

\( \sum_{k \in M} y_{ik} = 1 \)  \hspace{1cm} (2)

\( y_{ik} \geq \sum_{j \in N} z_{ijk} \)  \hspace{1cm} (3)

\( y_{jk} \geq \sum_{i \in N} z_{ijk} \)  \hspace{1cm} (4)

\( R_{jk} = a_j - ST_{k, j} = \min(j), j \in (j \mid y_{jk} = 1) \)  \hspace{1cm} (5)

\( R_{N+1,k} = ET_{k} - d_i, i = \max(i), i \in (i \mid y_{ik} = 1) \)  \hspace{1cm} (6)

\( R_{i,k} = a_i - d_i, \forall (i, j, k) \in \{(i, j, k) \mid z_{ijk} = 1\} \)  \hspace{1cm} (7)

\( a_j + (1 - z_{ijk})S \geq d_i + I \)  \hspace{1cm} (8)

\( v_i \leq e_k + (1 - y_{ik})S \)  \hspace{1cm} (9)

First of all, the index \( i, j, k \) in equation (1)-(9) denote \( i, j \in N, k \in M \); secondly, equation (1) denotes the objective function with the minimum overall variance of slack time, and \( \bar{R} \) in equation (1) is \( \sum_{k=1}^{M} (ET_k - ST_k) - \sum_{i=1}^{N} (d_i - a_i) \sqrt{N + M} \); finally, constraint (2) indicates that every flight must be assigned to one gate. Constraint (3) specifies every flight has one immediately proceeding flight at most. Constraint (4) specifies every flight has one immediately succeeding flight at most. Constraint (5) and (6) defines the first and last slack time of each gate, and constraint (7) states the others slack time. Constraint (8) stipulates the flight can be assigned to the gate when the preceding flight has departed for dwell time. Constraint (9) the different type of gate allow to park different type of flight.

III. TS Algorithm Design

There is no known algorithm for finding the optimal solution within a polynomial-bounded amount of time for model is NP-hard problem. With the increase of the problem dimension (as captured by the number of aircrafts and number of gates), its computational complexity increases exponentially. Hence, challenges in solving the problem efficiently are posed. Tabu Search (TS), introduced by Glover F. [14], has been widely used in optimization where it has been effective.

A. Initial Feasible Solution

AGAP is a discrete integer programming problem and NP-hard problem, so we design the meta-heuristic method to solve the problem, in other words, according to the meta-heuristic method, we have an initial airport gate assignment solution as initial feasible solution in TS to improve the convergence speed of TS algorithm. The process of meta-heuristic method is shown as follows:

According to the intrinsic characteristics of the objective function itself, consider a population slack time \( X_1, X_2, \cdots, X_E \) with size \( E \) and variance \( \sigma_E^2 \).

Suppose that the slack time of simple points \( X_1, X_2, \cdots, X_E \) are observed and the remaining simple points \( X_{e+1}, X_{e+2}, \cdots, X_E \) are to be realized later. The overall variance \( \sigma_E^2 \) is shown as follows:

\[
\sigma_E^2 = \frac{1}{E} \sum_{i=1}^{E} X_i^2 - \left( \frac{1}{E} \sum_{i=1}^{E} X_i \right)^2
\]

Let \( \mu_e = \sum_{i=1}^{e} X_i \), \( \zeta_e = \sum_{i=1}^{e} X_i^2 \). Equation (10) can be written by using \( \mu_e \) and \( \zeta_e \), so that

\[
\sigma_E^2 = \frac{\zeta_e + \sum_{i=e+1}^{E} X_i^2}{E} - \left( \frac{\mu_e + \sum_{i=e+1}^{E} X_i}{E} \right)^2
\]

The second order partial derivative in (11) states that

\[
\frac{\partial^2 \sigma_E^2}{\partial X_j} = \frac{2X_j}{E} - \frac{2 \left( \mu_e + \sum_{i=e+1}^{E} X_i \right)}{E^2}
\]

\[
\frac{\partial^2 \sigma_E^2}{\partial X_j^2} = \frac{2}{E} - \frac{2}{E^2} = \frac{2(E - 1)}{E^2}
\]

where \( j = e + 1, e + 2, \cdots, E \).
Obviously $\frac{\partial^2 \sigma^2}{\partial X_j^2} > 0$ for $E > 2$, so there is the minimum value in (11). Let $\frac{\partial \sigma^2}{\partial X_j} = 0$ in order to find the minimum value, that is

$$\frac{\partial \sigma^2}{\partial X_j} = \frac{2X_j - \frac{2}{E} \left( \mu_{\epsilon} + \sum_{i=1}^{E} X_i \right)}{E^2} = 0 \quad (14)$$

Notice that $X^*_j = \frac{\mu_{\epsilon}}{e}$ satisfies equation (14), so a lower bound on overall variance of $\sigma^2$ is given by

$$\sigma^2 \geq \frac{E \cdot \sigma^2}{E} \quad (15)$$

To make easier to understand in equation (15), that is

$$\sigma^2 \geq \frac{e \cdot \sigma^2}{e} \quad (16)$$

Where $\sigma^2 = \frac{\mu_{\epsilon}^2}{e} - \left( \frac{\mu_{\epsilon}}{e} \right)^2$.

We have computed the lower bound on overall variance of $\sigma^2$ from equation (10) to equation (16), so the initial feasible solution in TS is solved by the lower bound, and the basic details of the meta-heuristic method are as follows:

**Step1:** Set $\alpha = 0, \beta = 0, \theta_k = ST_k, \quad i = 1$, the number of slack time is $N + M$, so the evaluation indicator of AGAP can be determined by $f' = \frac{\beta - \frac{\alpha^2}{i}}{N + M}$.

**Step2:** If $i < N$, jump to step 3; otherwise output the solution.

**Step3:** Find $k^* = \arg \min_{k=1, 2, \ldots, M} a_i \theta_k^j$; update the decision variable $y_{ik} = 1$.

**Step4:** Set $\text{dwellTime} = a_i - \theta_k^j$, $\theta_k = d_j$, $\alpha = \alpha + \text{dwellTime}$, $\beta = \beta + \text{dwellTime}^2$.

**Step5:** $i = i + 1$, jump to step 2.

**B. Neighborhood Search Moves**

In [15], three kinds of neighborhood moves are proposed which are used in the tabu search algorithm to solve the classical AGAP. They are described as follows:

1) **Insert Move:** Move a single flight to a gate other than the one it currently assigns.

2) **Exchange I Move:** Exchange two flights and their gate assignments.

3) **Exchange II Move:** Exchange two flight pairs in the current assignment. The two flights in the flight pair have to be consecutive.

Those moves, however, are shown to be ineffective at times. In heuristics that are proposed in [12], an Interval Exchange Move is designed to replace the Exchange I Move and Exchange II Move. The Interval Exchange Move exchanges two flight intervals in the current assignment. A flight interval consists of one or more consecutive flights in one gate. The authors show that the tabu search which adopts Interval Exchange Move is superior than methods used in [15].

Here, we will use the Insert Move and Interval Exchange Move to formulate neighborhood search moves which are introduced briefly in the following sections. The details are not presented here due to the space limitation.

**The Insert Move** $\text{insert}(i, k) \rightarrow (i, l)$ moves a flight $i$, which is currently assigned to gate $k$, to gate $l (k \neq l)$. In each move, $i$ and $l$ are randomly selected. ($k$ is naturally obtained from the current solution.)

**The Interval Exchange Move** $(i, j, k) \leftrightarrow (i', j', l)$ exchanges the time durations from flights $i$ to $j$ in gate $k$ with flights $i'$ to $j'$ in gate $l$. The idea was proposed in [12] to overcome a weakness of Exchange I and Exchange II moves in [15].

**C. Tabu Search Framework**

The TS algorithm can be described by the following steps:

**Step1:** Generate an initial feasible solution $x_{\text{init}}$ by the meta-heuristic method, set $x_{\text{init}} \rightarrow x_{\text{curr}}$.

**Step2:** Generate a set of neighborhood solutions $N(x_{\text{curr}})$ of $x_{\text{curr}}$ by the Insert Move and Exchange Move.

**Step3:** The solution $x' \in N(x_{\text{curr}})$ with the least probability of conflict will be selected: (1) it is not forbidden (i.e. the assignment is not identical to any assignments of recent tabu tenure moves); (2) The objective value of $x'$ is better than the current best objective value.

**Step4:** Set $x' \rightarrow x_{\text{curr}}$; update the TS memory.

**Step5:** If the termination conditions are satisfied, stop; otherwise jump to step 2.

When we generate the neighborhood solutions, we randomly choose Insert Move and Interval Exchange Move with equal probability. There are two termination conditions: either the best solution cannot be improved within a certain number of iterations, or the maximum number of iterations has been reached.

**IV. EXPERIMENTAL RESULTS**

We conducted elaborate experiments to compare our TS heuristic with other such approaches, specifically, with initial
solution from the approach of [16]. There is one objective to minimize the overall variance of slack times, whereas we are interested, additionally, the initial solution is given by random algorithm in [16]. More importantly, we incorporates additional constraints such as the type of gate and aircraft, then we focus on TS heuristics for application to the AGAP and it is this approach that we make comparisons with here.

Based on the test data for one day schedule of flights assigned to gates of one terminal in Beijing International airport. There are 10 gates in this terminal and there are 100 flights need gates from 06:00 to 16:00. To illustrate the TS heuristic efficiently in this paper and compare with [16], thereby, the parameters of TS are set as follows:

- Iteration number is 30
- Tabu Neighborhood number is 3
- Tabu Search memory length is $\sqrt{M}$.

The test data are calculated thirty times and the best value of objective is 9821 with the meta-heuristic method as the initial feasible solution in TS, but the best value of objective is 15775 with the random algorithm in [16] as the initial feasible solution in TS. Obviously, in the Fig 1, we can think that the effect of optimization is better than random algorithm in [16]. The experimental results are shown in Fig 1.

![Figure 1. The Process of TS](image)

Figure 1. The Process of TS

From the above analysis, we can draw some conclusive remarks as follows:

1) The meta-heuristic method and TS algorithm may perform well in optimization because they provide a lower bound in this model, and the initial feasible solution is given in polynomial-time.

2) The shortest slack time is 5 minutes. The longest slack time is 70 minutes. The number of slack times that is longer than 45 minutes is 9, where the beginning and close slack times of gates were not considered. The distribution of slack time is shown in Fig 2.

![Figure 2. Slack Time Distribution](image)

Figure 2. Slack Time Distribution

V. CONCLUSION

In this paper, we have proposed an AGAP model, which is to minimize the overall variance of slack times. In order to solve the model more conveniently, we provided a meta-heuristic method that can allocate the flights to minimize the objective function as well as to provide an initial feasible solution. We then used a TS algorithm to optimize the AGAP. Experiments were conducted using a range of test data sets with the real-life, the results shown that our algorithm is compared with a previous method, adapted for the model, and the algorithms are efficient.

Future improvements of our meta-heuristic TS approach are anticipated to result by using more refined candidate list strategies and other intensification and diversification strategies. Our model should incorporate various practical constraints and refined objective functions. Finally, the current AGAP model and algorithm is devised for the planning problem. To be a successful implementation in real world, our model and algorithm must be extended to handle the real-time gate assignment by patching the disrupted assignment caused by operations such as flight delays and cancellations.

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REFERENCES


